The Circle (Q 3, Paper 2)

Lesson No. 1: The Simple Circle

2007

3 (a) The circle C has centre (0, 0) and radius 4.

(i) Write down the equation of C.

(ii) Verify that the point (3, 2) lies inside the circle C.

Solution

3 (a) (i)

Centre (0, 0), $r = 4$

Circle C: $x^2 + y^2 = 4^2 \Rightarrow x^2 + y^2 = 16$

Circle C with centre (0, 0), radius r.

$x^2 + y^2 = r^2$ 1

3 (a) (ii)

Is a point on a circle, inside a circle or outside a circle?
Substitute the point into the circle.
On the circle: Both sides are equal.
Inside the circle: The left hand side is less than the right hand side.
Outside the circle: The left hand side is greater than the right hand side.

To show (3, 2) is inside C:

$(3)^2 + (2)^2 = 9 + 4 = 13 < 16 \Rightarrow (3, 2)$ is inside C.

2005

3 (a) The circle C has equation $x^2 + y^2 = 49$.

(i) Write down the centre and the radius of C.

(ii) Verify that the point $(5, -5)$ lies outside the circle C.

Solution

3 (a) (i)

Centre (0, 0), radius $r = \sqrt{49} = 7$

Circle C with centre (0, 0), radius r.

$x^2 + y^2 = r^2$ 1

3 (a) (ii)

Is a point on a circle, inside a circle or outside a circle?
Substitute the point into the circle.
On the circle: Both sides are equal.
Inside the circle: The left hand side is less than the right hand side.
Outside the circle: The left hand side is greater than the right hand side.

To show that $(5, -5)$ lies outside C:

$C : x^2 + y^2 = 49$

$\Rightarrow (5)^2 + (-5)^2 = 25 + 25$

$= 50 > 49 \Rightarrow (5, -5)$ lies outside C.

2004

3 (a) The circle C has equation $x^2 + y^2 = 36$.

(i) Write down the radius of C.

(ii) The radius of another circle is twice the radius of C.
The centre of this circle is (0, 0). Write down its equation.

SOLUTION

3 (a) (i)

$x^2 + y^2 = 36 \Rightarrow r = \sqrt{36} = 6$

Circle C with centre (0, 0), radius r.

$x^2 + y^2 = r^2$ 1

3 (a) (ii)

New circle: centre (0, 0), $r = 12$

$x^2 + y^2 = 12^2 \Rightarrow x^2 + y^2 = 144$

2003

3 (a) The circle C has equation $x^2 + y^2 = 25$.

(i) Verify that the point $(-4, 3)$ is on the circle C.

(ii) Write down the coordinates of a point that lies outside C and give a reason for your answer.

SOLUTION

3 (a) (i)

IS A POINT ON A CIRCLE, INSIDE A CIRCLE OR OUTSIDE A CIRCLE?
Substitute the point into the circle.
On the circle: Both sides are equal.
Inside the circle: The left hand side is less than the right hand side.
Outside the circle: The left hand side is greater than the right hand side.

$(-4, 3) \in x^2 + y^2 = 25?$

$(-4)^2 + (3)^2 = 16 + 9$

$= 25 \Rightarrow (-4, 3) \in x^2 + y^2 = 25$

3 (a) (ii)

You need to pick a value of x and a value of y such that when you put it into the equation of the circle the left hand side is greater than 25.

(4, 5) is such a number because $(4)^2 + (5)^2 = 16 + 25 = 41 > 25$.

2002

3 (a) Write down the coordinates of any three points that lie on the circle with equation $x^2 + y^2 = 100$.

SOLUTION

IS A POINT ON A CIRCLE, INSIDE A CIRCLE OR OUTSIDE A CIRCLE?
Substitute the point into the circle.
On the circle: Both sides are equal.
Inside the circle: The left hand side is less than the right hand side.
Outside the circle: The left hand side is greater than the right hand side.

You need to pick values of x and y which when you put them into the equation you get 100.

(6, 8) is a point on the circle because $(6)^2 + (8)^2 = 36 + 64 = 100$.

(8, 6) is a point on the circle because $(8)^2 + (6)^2 = 64 + 36 = 100$.

(10, 0) is a point on the circle because $(10)^2 + (0)^2 = 100 + 0 = 100$.

2001

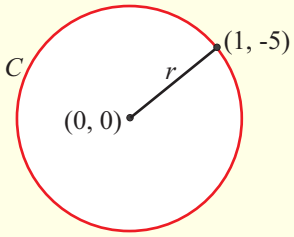
3 (c) C is a circle with centre (0, 0). It passes through the point $(1, -5)$.

(i) Write down the equation of C.

(ii) The point (p, p) lies inside C where $p \in \mathbf{Z}$.
Find all the possible values of p.

SOLUTION

3 (c) (i)



Circle C with centre (0, 0), radius r.

$x^2 + y^2 = r^2$ 1

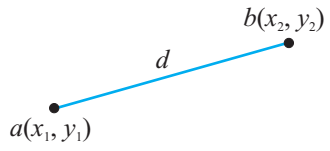
Find the radius by finding the distance between the centre (0, 0) and the point on the circle $(1, -5)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ 1

The distance between a and b is written as $|ab|$.

REMEMBER THE DISTANCE FORMULA AS:

$d = \sqrt{(\text{Difference in } x\text{'s})^2 + (\text{Difference in } y\text{'s})^2}$



$(0, 0) \quad (1, -5)$
$\downarrow\downarrow \quad \downarrow \quad \downarrow$
$x_1 \; y_1 \quad x_2 \; y_2$

$r = \sqrt{(1-0)^2 + (-5-0)^2}$

$\Rightarrow r = \sqrt{1^2 + (-5)^2} = \sqrt{1 + 25}$

$\therefore r = \sqrt{26}$

Equation of C: Centre (0, 0), $r = \sqrt{26}$

$C : x^2 + y^2 = (\sqrt{26})^2 \Rightarrow x^2 + y^2 = 26$

3 (c) (ii)

Do this by inspection. p is an integer which is a whole number (positive and negative.)

> **IS A POINT ON A CIRCLE, INSIDE A CIRCLE OR OUTSIDE A CIRCLE?**
> Substitute the point into the circle.
> **On the circle**: Both sides are equal.
> **Inside the circle**: The left hand side is less than the right hand side.
> **Outside the circle**: The left hand side is greater than the right hand side.

$p = 0 : (0,\ 0) \Rightarrow (0)^2 + (0)^2 = 0 < 26 \Rightarrow (0,\ 0)$ is inside the circle.

$p = 1 : (1,\ 1) \Rightarrow (1)^2 + (1)^2 = 1 + 1 = 2 < 26 \Rightarrow (1,\ 1)$ is inside the circle.

$p = -1 : (-1,\ -1) \Rightarrow (-1)^2 + (-1)^2 = 1 + 1 = 2 < 26 \Rightarrow (-1,\ -1)$ is inside the circle.

$p = 2 : (2,\ 2) \Rightarrow (2)^2 + (2)^2 = 4 + 4 = 8 < 26 \Rightarrow (2,\ 2)$ is inside the circle.

$p = -2 : (-2,\ -2) \Rightarrow (-2)^2 + (-2)^2 = 4 + 4 = 8 < 26 \Rightarrow (-2,\ -2)$ is inside the circle.

$p = 3 : (3,\ 3) \Rightarrow (3)^2 + (3)^2 = 9 + 9 = 18 < 26 \Rightarrow (3,\ 3)$ is inside the circle.

$p = -3 : (-3,\ -3) \Rightarrow (-3)^2 + (-3)^2 = 9 + 9 = 18 < 26 \Rightarrow (-3,\ -3)$ is inside the circle.

$p = 4 : (4,\ 4) \Rightarrow (4)^2 + (4)^2 = 16 + 16 = 32 > 26 \Rightarrow (4,\ 4)$ is outside the circle.

$p = -4 : (-4,\ -4) \Rightarrow (-4)^2 + (-4)^2 = 16 + 16 = 32 > 26 \Rightarrow (-4,\ -4)$ is outside the circle.

All whole numbers of p between -3 and 3 give rise to points inside the circle.

ANOTHER WAY:

Find out the values of p for which (p, p) is on the circle.

$(p,\ p) \in x^2 + y^2 = 26 \Rightarrow (p)^2 + (p)^2 = 26$

$\Rightarrow 2p^2 = 26$

$\Rightarrow p^2 = 13$

$\therefore p = \pm\sqrt{13} \approx \pm 3.6$

Therefore, the point (p, p) is inside the circle for values of p between $-\sqrt{13}$ and $\sqrt{13}$.

Therefore, the point (p, p) is inside the circle for whole number values of p between -3 and 3.

ANS: $p = \{-3, -2, -1, 0, 1, 2, 3\}$

2000

3 (a) The circle C has equation $x^2 + y^2 = 16$.

(i) Write down the length of the radius of C.

(ii) Show, by calculation, that the point (3, 1) is inside the circle.

SOLUTION

3 (a) (i)

$C : x^2 + y^2 = 16$

$\Rightarrow r = \sqrt{16} = 4$

Circle C with centre (0, 0), radius r.

$x^2 + y^2 = r^2$ 1

3 (a) (ii)

IS A POINT ON A CIRCLE, INSIDE A CIRCLE OR OUTSIDE A CIRCLE?
Substitute the point into the circle.
On the circle: Both sides are equal.
Inside the circle: The left hand side is less than the right hand side.
Outside the circle: The left hand side is greater than the right hand side.

$(3)^2 + (1)^2 = 9 + 1$

$= 10 < 16 \Rightarrow (3, 1)$ is inside the circle.

1999

3 (a) C is a circle with centre (0, 0) passing through the point (8, 6).
Find

(i) the radius length of C

(ii) the equation of C.

SOLUTION

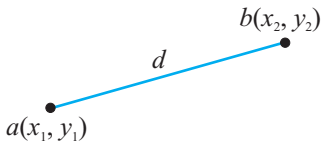
3 (a) (i)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ 1

The distance between a and b is written as $|ab|$.

REMEMBER THE DISTANCE FORMULA AS:

$d = \sqrt{(\text{Difference in } x\text{'s})^2 + (\text{Difference in } y\text{'s})^2}$

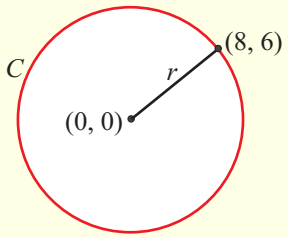


(0, 0) (8, 6)
x_1 y_1 x_2 y_2

$r = \sqrt{(8-0)^2 + (6-0)^2}$

$\Rightarrow r = \sqrt{8^2 + 6^2} = \sqrt{64 + 36}$

$\therefore r = \sqrt{100} = 10$



3 (a) (ii)

Equation of C: centre (0, 0), r = 10

$C : x^2 + y^2 = 100$

Circle C with centre (0, 0), radius r.

$x^2 + y^2 = r^2$ 1

1998

3 (a) A circle C, with centre $(0, 0)$, passes through the point $(4, -3)$.

(i) Find the length of the radius of C.

(ii) Show, by calculation, that the point $(6, -1)$ lies outside C.

SOLUTION

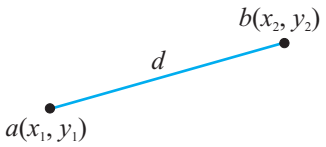
3 (a) (i)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \ldots\ldots \textbf{1}$$

The distance between a and b is written as $|ab|$.

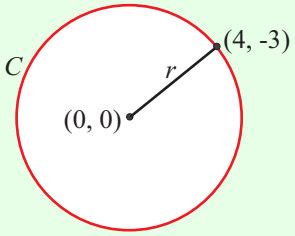
REMEMBER THE DISTANCE FORMULA AS:

$$d = \sqrt{(\text{Difference in } x\text{'s})^2 + (\text{Difference in } y\text{'s})^2}$$



$$\begin{array}{cc} (0,\ 0) & (4,\ -3) \\ \downarrow\downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ \ y_2 \end{array}$$

$$r = \sqrt{(4-0)^2 + (-3-0)^2}$$
$$\Rightarrow r = \sqrt{(4)^2 + (-3)^2} = \sqrt{16+9}$$
$$\therefore r = \sqrt{25} = 5$$



3 (a) (ii)

You can show that the distance d from the centre $(0, 0)$ to $(6, -1)$ is greater than the radius.

$$\begin{array}{cc} (0,\ 0) & (6,\ -1) \\ \downarrow\downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ \ y_2 \end{array}$$

$$d = \sqrt{(6-0)^2 + (-1-0)^2}$$
$$\Rightarrow d = \sqrt{(6)^2 + (-1)^2} = \sqrt{36+1}$$
$$\therefore d = \sqrt{37}$$

$d > r$ as $\sqrt{37} > \sqrt{25}$.

1996

3 (a) The equation of a circle is $x^2 + y^2 = 36$.

(i) Write down its radius length.

(ii) Verify, by calculation, that the point (2, 3) is inside the circle.

SOLUTION

3 (a) (i)

$x^2 + y^2 = 36$

$\Rightarrow r = \sqrt{36} = 6$

Circle C with centre (0, 0), radius r.

$x^2 + y^2 = r^2$ 1

3 (a) (ii)

IS A POINT ON A CIRCLE, INSIDE A CIRCLE OR OUTSIDE A CIRCLE?
Substitute the point into the circle.
On the circle: Both sides are equal.
Inside the circle: The left hand side is less than the right hand side.
Outside the circle: The left hand side is greater than the right hand side.

$(2, 3): x^2 + y^2 = 36$

$\Rightarrow (2)^2 + (3)^2 = 4 + 9$

$= 13 < 36 \Rightarrow (2, 3)$ is inside the circle.