## The Circle (Q 3, Paper 2)

## 2011

3. (a) A circle has equation $x^{2}+y^{2}=81$.
(i) Write down the co-ordinates of the centre of the circle.
(ii) Find the radius of the circle.
(b) The circle $c$ has equation $(x-3)^{2}+(y+1)^{2}=17$.
(i) Verify that the point $(7,-2)$ is on $c$.
(ii) On a co-ordinate diagram, mark the centre of $c$ and draw $c$.
(iii) Find, using algebra, the co-ordinates of the two points at which $c$ intersects the $x$-axis.
(c) The points $A(-1,2), B(-3,-4), C(3,-6)$ and $D(5,0)$ are the vertices of a square. The sides of the square are tangents to the circle $s$, as shown.
(i) Find the co-ordinates of the centre of $s$.
(ii) Find the equation of $s$.
(iii) The circle $(x+4)^{2}+y^{2}=10$ is the image of $s$ under the translation $(p, q) \rightarrow(6,5)$.
 Find the value of $p$ and the value of $q$.

## Solution

3 (a) (i) Circle $c$ with centre $(0,0)$, radius $r . x^{2}+y^{2}=r^{2}$
$c: x^{2}+y^{2}=81$
Centre (0, 0)
3 (a) (ii)
$r^{2}=81 \Rightarrow r=\sqrt{81}=9$

3 (b) (i)
Is a point on a circle, inside a circle or outside a circle?
Substitute the point into the circle.
On the circle: Both sides are equal.
Inside the circle: The left hand side is less than the right hand side.
Outside the circle: The left hand side is greater than the right hand side.
$(7,-2) \in(x-3)^{2}+(y+1)^{2}=17 ?$
$(7-3)^{2}+(-2+1)^{2}$
$=(4)^{2}+(-1)^{2}$
$=16+1$
$=17$
Therefore ( $7,-2$ ) is on the circle.
3 (b) (ii)


3 (b) (iii)
To find out where the circle, $C$, crosses the $x$-axis: Set $y=0$ in the circle equation.
To find out where the circle, $C$, crosses the $y$-axis:
Set $x=0$ in the circle equation.
$y=0:(x-3)^{2}+(0+1)^{2}=17$
$(x-3)^{2}+(1)^{2}=17$
$(x-3)^{2}+1=17$
$(x-3)^{2}=17-1$
$(x-3)^{2}=16$
$(x-3)= \pm \sqrt{16}$
$(x-3)= \pm 4$
Solve each equation separately.

$$
\begin{array}{l|l}
x-3=-4 & x-3=4 \\
x=-4+3 & x=4+3 \\
\therefore x=-1 & \therefore x=7
\end{array}
$$

The $x$-intercepts are $(7,0)$ and $(-1,0)$.

## 3 (c) (i)

Vertices of square: $A(-1,2), B(-3,-4), C(3,-6), D(5,0)$
Call $O$ the centre of circle $s$.
$O$ is the midpoint of $[A C]$.

$$
\text { Midpoint }=\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)
$$

$$
\left\lvert\, \begin{array}{ccc}
A(-1,2) & C(3,-6) \\
\downarrow & \downarrow & \downarrow \\
x_{1} y_{1} & x_{2} & y_{2}
\end{array}\right.
$$

Midpoint $=\left(\frac{-1+3}{2}, \frac{2-6}{2}\right)=\left(\frac{2}{2}, \frac{-4}{2}\right)=(1,-2)$
3 (c) (ii) Circle $c$ with centre $(h, k)$, radius $r . \quad(x-h)^{2}+(y-k)^{2}=r^{2}$

To find the equation of $s$ you need the centre and radius $r$.

$$
\begin{aligned}
& r=\frac{1}{2}|A B| \begin{array}{ccc}
A(-1,2) & B(-3,-4) \\
\downarrow & \downarrow & \downarrow \\
x_{1} y_{1} & x_{2} & y_{2}
\end{array} \\
& \\
& \qquad \begin{aligned}
&|A B|=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}} \\
&=\sqrt{(-3-(-1))^{2}+(-4-2)^{2}} \\
&=\sqrt{(-3+1)^{2}+(-4-2)^{2}} \\
&=\sqrt{(-2)^{2}+(-6)^{2}} \\
&=\sqrt{40} \\
& \quad=\sqrt{4 \times 10} \\
& \quad=2 \sqrt{10}
\end{aligned} \\
& \begin{aligned}
& \therefore r=\frac{1}{2}|A B|=\sqrt{10} \\
& \text { Equation of } s: \text { Centre }(1,-2), r=\sqrt{10} \\
& s:(x-1)^{2}+(y-(-2))^{2}=(\sqrt{10})^{2} \\
&(x-1)^{2}+(y+2)^{2}=10
\end{aligned}
\end{aligned}
$$

## 3 (c) (iii)

Circle: $(x+4)^{2}+y^{2}=10$
Centre: $(-4,0)$
To go from $O$ to $(-4,0)$, you need to subtract 5 from the $x$-coordinate and add 2 to the $y$-coordinate.
Do the same to go from $(p, q)$ to $(6,5)$.
$p-5=6 \Rightarrow p=6+5=11$
$q+2=5 \Rightarrow q=5-2=3$


