The Circle (Q 3, Paper 2)

2010

3 (a) A circle with centre (0, 0) passes through the point (5, –12).

(i) Find the radius of the circle.

(ii) Write down the equation of the circle.

(b) The circle c has equation $x^2 + y^2 = 17$.
l is the line $x - 4y - 17 = 0$.
The line l is a tangent to c at the point T.

(i) Find the co-ordinates of T.

(ii) The point T is one end-point of a diameter of c.
Find the co-ordinates of the other end-point.

(c) A circle has equation $x^2 + (y-7)^2 = 100$.

(i) Write down the co-ordinates of the centre of the circle and the radius of the circle.

(ii) The point $(6, h)$ is on the circle. Find the two possible values of h.

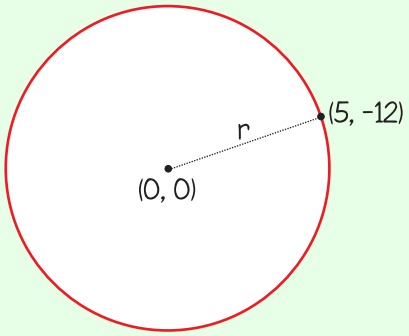
Solution

3 (a) (i)

$$\begin{matrix}(0,\ 0) & (5,\ -12)\\ \downarrow\ \downarrow & \downarrow\ \ \downarrow\\ x_1\ y_1 & x_2\ \ y_2\end{matrix}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$r = \sqrt{(5-0)^2 + (-12-0)^2}$$
$$= \sqrt{5^2 + (-12)^2}$$
$$= \sqrt{25 + 144}$$
$$= \sqrt{169} = 13$$



3 (a) (ii)

Equation of circle: Centre (0, 0), $r = 13$

$$x^2 + y^2 = 13^2$$
$$x^2 + y^2 = 169$$

Circle C with centre (0, 0), radius r.

$$x^2 + y^2 = r^2$$

3 (b) (i)

FINDING THE POINT OF CONTACT BETWEEN A TANGENT AND A CIRCLE:

STEPS
1. Isolate x or y using equation of the line.
2. Substitute into the equation of the circle and solve the resulting quadratic.

$l: x-4y-17=0 \Rightarrow x=4y+17$

$c: x^2+y^2=17$

$(4y+17)^2+y^2=17$

$(4y+17)(4y+17)+y^2=17$

$(16y^2+68y+68y+289)+y^2=17$

$17y^2+136y+272=0$

$y^2+8y+16=0$

$(y+4)(y+4)=0$

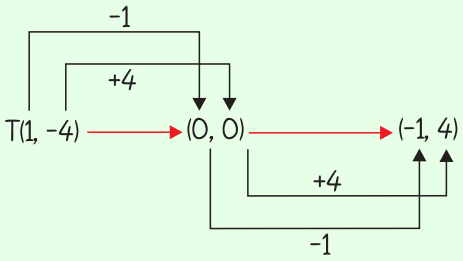
$y=-4$

$\therefore x=4y+17=4(-4)+17=-16+17=1$

$\therefore T(1,-4)$

3 (b) (ii)

Pass the point T through the centre (0, 0) by a central symmetry to get a point of (–1, 4).



3 (c) (i)

$x^2+(y-7)^2=100$

$(x-0)^2+(y-7)^2=10^2$

Centre: $(0,\ 7),\ r=10$

Circle C with centre (h, k), radius r.

$(x-h)^2+(y-k)^2=r^2$

3 (c) (ii)

$(6,\ h) \in x^2+(y-7)^2=100$

$\Rightarrow 6^2+(h-7)^2=100$

$36+(h-7)^2=100$

$(h-7)^2=64$

$h-7=\sqrt{64}=\pm 8$

$h-7=8$	$h-7=-8$
$\therefore h=15$	$\therefore h=-1$

IS A POINT ON A CIRCLE?
Substitute the point into the circle.
On the circle: Both sides are equal.