## The Circle (Q 3, Paper 2)

2009
3 (a) The circle $C$ has equation $x^{2}+y^{2}=25$.
(i) Write down the radius of $C$.
(ii) Verify that the point $(4,-3)$ is on $C$.
(iii) The line $T$ is a tangent to $C$ at the point $(4,-3)$. Find the equation of $T$.
(iv) On a co-ordinate diagram, draw the circle $C$ and the tangent $T$.
(v) $L$ is a tangent to $C$ and $L$ is parallel to the $x$-axis. Find the two possible equations of $L$.
(b) The point $c(1,-6)$ is the centre of the circle $K$, as shown.
The point $r(9,0)$ is on the circle.
(i) Find the radius of the circle.
(ii) Write down the equation of the circle.


The vertices of the rectangle $r s t u$ are on the circle and $s r$ is horizontal.
(iii) Find the co-ordinates of $t$, the co-ordinates of $s$ and the co-ordinates of $u$.

## Solution

3 (a) (i)
$x^{2}+y^{2}=25$
$x^{2}+y^{2}=5^{2}$
$\therefore r=5$

Circle $C$ with centre ( 0,0 ), radius $r$.
$x^{2}+y^{2}=r^{2}$

3 (a) (ii)
$(4,-3) \in C$ ?
Is a Point on a circle?
Substitute the point into the circle.
On the circle: Both sides are equal.
$=(4)^{2}+(-3)^{2}$
$=16+9$
$=25$
$\therefore(4,-3) \in C$

## 3 (a) (iii)

## Steps

1. Find the slope of the line from the centre to the point of contact.
2. Find the slope of the tangent (it is perpendicular to the radius).
3. Find the equation of $T$.

$$
\begin{array}{cccc}
(0,0) & (4, & -3) \\
\downarrow & \downarrow & \downarrow & \downarrow \\
x_{1} & y_{1} & x_{2} & y_{2}
\end{array} \quad \quad m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}
$$



Slope of line joining $(0,0)$ and $(4,-3): m=\frac{0-(-3)}{0-4}=\frac{3}{-4}=-\frac{3}{4}$
Slope of $T$ (perpendicular slope): $m=\frac{4}{3}$
Equation of $T$ : Point $(4,-3), m=\frac{4}{3}$

$$
\begin{aligned}
& y-(-3)=\frac{4}{3}(x-4) \quad y-y_{1}=m\left(x-x_{1}\right) \\
& y+3=\frac{4}{3}(x-4) \\
& 3(y+3)=4(x-4) \\
& 3 y+9=4 x-16 \\
& 0=4 x-3 y-25
\end{aligned}
$$

## 3 (a) (iv)



3 (a) (v)


## 3 (b) (i)

Radius: $|c r|=\sqrt{(9-1)^{2}+(0-(-6))^{2}} \quad d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$

$$
\begin{aligned}
& =\sqrt{8^{2}+6^{2}} \\
& =\sqrt{64+36} \\
& =\sqrt{100}=10
\end{aligned}
$$

## 3 (b) (ii)

Equation of $K$ : Centre (1, -6 ), $r=10$
$(x-1)^{2}+(y-(-6))^{2}=10^{2}$

Circle $C$ with centre $(h, k)$, radius $r$.
$(x-h)^{2}+(y-k)^{2}=r^{2}$
$(x-1)^{2}+(y+6)^{2}=100$

## 3 (b) (iii)

Pass the point $r$ through $c$ by a central symmetry. $r(9,0) \rightarrow c(1,-6) \rightarrow t(-7,-12)$


The other points, $s$ and $u$, are found by inspection.


