THE CIRCLE (Q 3, PAPER 2)

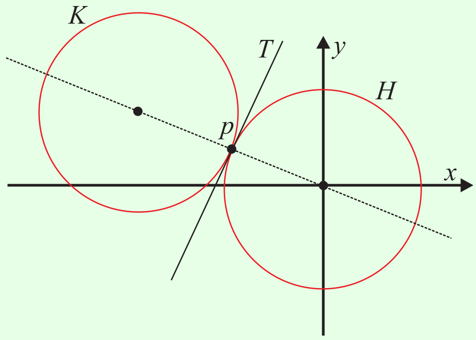
2008

3 (a) A circle has equation $x^2 + y^2 = 16$.

(i) Show the circle on a co-ordinate diagram.

(ii) Mark the four points at which the circle intersects the axes and label them with their co-ordinates.

(b) The diagram shows two circles H and K, of equal radius.
The circles touch at the point $p(-2, 1)$.
The circle H has centre $(0, 0)$.



(i) Find the equation of H.

(ii) Find the equation of K.

(iii) T is a tangent to the circles at p.
Find the equation of T.

(c) The circle S has equation $(x-3)^2 + (y+2)^2 = 40$.
S intersects the x-axis at the point a and at the point b.

(i) Find the co-ordinates of a and the co-ordinates of b.

(ii) Show that $|ab|$ is less than the diameter of S.

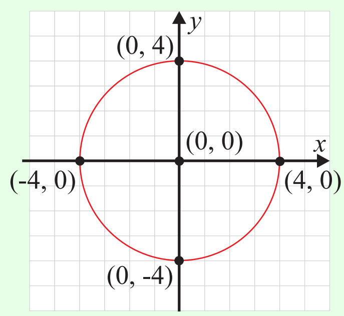
(iii) Find the equation of the circle with $[ab]$ as diameter.

SOLUTION

3 (a)

Circle C with centre $(0, 0)$, radius r.

$\boxed{x^2 + y^2 = r^2}$ 1



$x^2 + y^2 = 16$

Centre $(0, 0)$, $r = 4$

> TO FIND OUT WHERE THE CIRCLE, C, CROSSES THE x-AXIS:
> Set $y = 0$ in the circle equation.
> TO FIND OUT WHERE THE CIRCLE, C, CROSSES THE y-AXIS:
> Set $x = 0$ in the circle equation.

x-intercepts: Put $y = 0 \Rightarrow x^2 + (0)^2 = 16 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

$\therefore (4, 0), (-4, 0)$ are the x-intercepts.

y-intercepts: Put $x = 0 \Rightarrow (0)^2 + y^2 = 16 \Rightarrow y^2 = 16 \Rightarrow y = \pm 4$

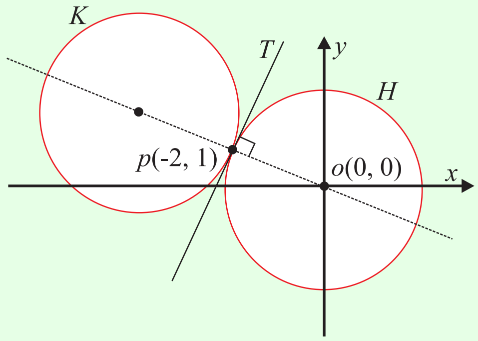
$\therefore (0, 4), (0, -4)$ are the y-intercepts.

3 (b) (i)

Equation of H: Centre (0, 0)

$r = |op| = \sqrt{(-2-0)^2 + (1-0)^2} = \sqrt{4+1}$

$\therefore r = \sqrt{5}$



Circle C with centre (0, 0), radius r.

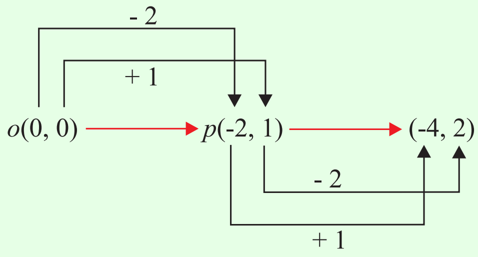
$\boxed{x^2 + y^2 = r^2}$ 1

Equation of H: $x^2 + y^2 = (\sqrt{5})^2 \Rightarrow x^2 + y^2 = 5$

3 (b) (ii)

To find the centre of K pass (0, 0) through p by a central symmetry.

Equation of K: $r = \sqrt{5}$, centre $(-4, 2)$



Circle C with centre (h, k), radius r.

$\boxed{(x-h)^2 + (y-k)^2 = r^2}$ 2

$K: (x-(-4))^2 + (y-2)^2 = (\sqrt{5})^2$

$\therefore K: (x+4)^2 + (y-2)^2 = 5$

3 (b) (iii)

T is perpendicular to op. Find the slope of op and hence, get the slope of the tangent T.

Slope of op: $m = \dfrac{1-0}{-2-0} = \dfrac{1}{-2} = -\dfrac{1}{2}$

$$\boxed{\begin{array}{cc} o(0,\ 0) & p(-2,\ 1) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ \ y_1 & x_2\ \ y_2 \end{array}}$$

$\boxed{m = \dfrac{y_2 - y_1}{x_2 - x_1}}$ 3

Slope of tangent T: $m = 2$

To form the equation of T, you need the slope ($m = 2$) and a point on T.

Use $p(-4, 2) = (x_1, y_1)$.

FINDING THE PERPENDICULAR SLOPE:
Invert the slope and change its sign.

$T: y - 1 = 2(x-(-2))$

$\Rightarrow T: y - 1 = 2(x+2)$

$\Rightarrow T: y - 1 = 2x + 4$

$\therefore T: 2x - y + 5 = 0$

Equation of a line: $\boxed{y - y_1 = m(x - x_1)}$ 4

3 (c) (i)

TO FIND OUT WHERE THE CIRCLE, C, CROSSES THE x-AXIS:
Set $y = 0$ in the circle equation.

$S: (x-3)^2 + (y+2)^2 = 40$

$y = 0 \Rightarrow (x-3)^2 + (0+2)^2 = 40$

$\Rightarrow (x-3)^2 + 4 = 40$

$\Rightarrow (x-3)^2 = 36$

$\Rightarrow (x-3) = \pm 6$

$\therefore x = -3, 9$

$\therefore a(-3, 0)$ and $b(9, 0)$ are the x-intercepts.

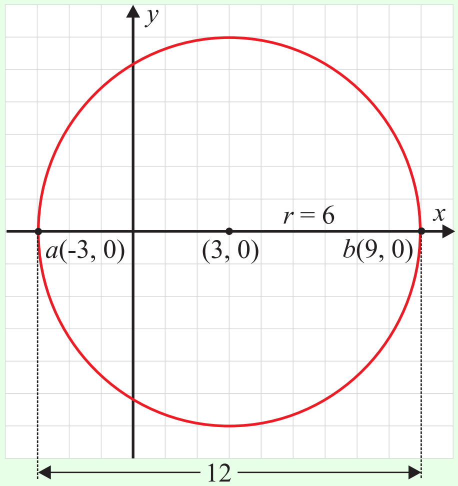
3 (c) (ii)

You can see from the diagram that $|ab| = 12$.

Radius of S: $r = \sqrt{40}$

Diameter of S: $d = 2\sqrt{40} \approx 12.65$

$$\Rightarrow |ab| < d$$



3 (c) (iii)

Equation of Circle: centre (3, 0), $r = 6$

Circle C with centre (h, k), radius r.

$\boxed{(x-h)^2 + (y-k)^2 = r^2}$ **2**

$(x-3)^2 + (y-0)^2 = 6^2$

$\Rightarrow (x-3)^2 + y^2 = 36$