THE CIRCLE (Q 3, PAPER 2)

2007

3 (a) The circle C has centre (0, 0) and radius 4.

(i) Write down the equation of C.

(ii) Verify that the point (3, 2) lies inside the circle C.

(b) The line $x - 3y = 0$ intersects the circle $x^2 + y^2 = 10$ at the points a and b.

(i) Find the coordinates of a and the coordinates of b.

(ii) Show that [ab] is a diameter of the circle.

(c) The circle K has equation $(x-5)^2 + (y+1)^2 = 34$.

(i) Write down the radius of K and the coordinates of the centre of K.

(ii) Verify that the point $(10, -4)$ is on the circle.

(iii) T is a tangent to the circle at the point $(10, -4)$.
S is another tangent to the circle and S is parallel to T.
Find the coordinates of the point at which S is a tangent to the circle.

SOLUTION

3 (a) (i)

Centre (0, 0), $r = 4$

Circle C: $x^2 + y^2 = 4^2 \Rightarrow x^2 + y^2 = 16$

> Circle C with centre (0, 0), radius r.
>
> $\boxed{x^2 + y^2 = r^2}$ 1

3 (a) (ii)

> **IS A POINT ON A CIRCLE, INSIDE A CIRCLE OR OUTSIDE A CIRCLE?**
> Substitute the point into the circle.
> **On the circle**: Both sides are equal.
> **Inside the circle**: The left hand side is less than the right hand side.
> **Outside the circle**: The left hand side is greater than the right hand side.

To show (3, 2) is inside C:

$(3)^2 + (2)^2 = 9 + 4 = 13 < 16 \Rightarrow (3, 2)$ is inside C.

3 (b) (i)

1. $L: x - 3y = 0 \Rightarrow x = 3y$

2. $C: x^2 + y^2 = 10$

$\Rightarrow (3y)^2 + y^2 = 10$

$\Rightarrow 9y^2 + y^2 = 10$

$\Rightarrow 10y^2 = 10 \Rightarrow y^2 = 1$

$\therefore y = \sqrt{1} = 1, -1$

$\therefore x = 3, -3$

Points of intersection: $a(-3, -1)$, $b(3, 1)$

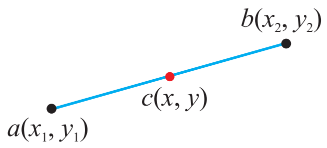
> **STEPS**
> **1.** Isolate x or y using equation of the line.
> **2.** Substitute into the equation of the circle and solve the resulting quadratic.

3 (b) (ii)

TO SHOW A LINE SEGMENT IS A DIAMETER OF A CIRCLE:
The midpoint of a diameter is the centre of a circle.

The formula for the midpoint, c, of the line segment $[ab]$ is:

$$\boxed{\text{Midpoint } = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)} \ \ldots\ldots\ \textbf{2}$$



REMEMBER THE MIDPOINT FORMULA AS: $\text{Midpoint } = \left(\frac{\text{Add the } x\text{'s}}{2}, \frac{\text{Add the } y\text{'s}}{2}\right)$

$$\begin{array}{cccc} a(-1, & -3) & b(1, & 3) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

$$\text{Midpoint of } [ab] = \left(\frac{-1+1}{2}, \frac{-3+3}{2}\right) = \left(\frac{0}{2}, \frac{0}{2}\right) = (0, 0)$$

Equation of Circle: $x^2 + y^2 = 10$
Centre $(0, 0)$

Circle C with centre $(0, 0)$, radius r.

$$\boxed{x^2 + y^2 = r^2} \ \ldots\ldots\ \textbf{1}$$

Therefore, $[ab]$ is the diameter of the circle as its midpoint is the centre of the circle.

3 (c) (i)

Circle C with centre (h, k), radius r.

$$\boxed{(x-h)^2 + (y-k)^2 = r^2} \ \ldots\ldots\ \textbf{2}$$

To get the centre: Change the sign of the number inside each bracket.
To get the radius: Take the square root of the number on the right.

$K : (x-5)^2 + (y+1)^2 = 34$

Centre $(5, -1)$, $r = \sqrt{34}$

3 (c) (ii)

IS A POINT ON A CIRCLE, INSIDE A CIRCLE OR OUTSIDE A CIRCLE?
Substitute the point into the circle.
On the circle: Both sides are equal.
Inside the circle: The left hand side is less than the right hand side.
Outside the circle: The left hand side is greater than the right hand side.

$(10, -4) \in K$?

$\therefore ((10)-5)^2 + ((-4)+1)^2$

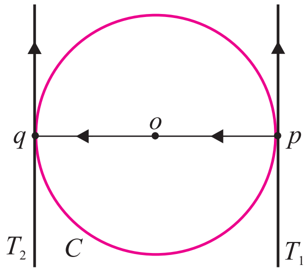
$= (10-5)^2 + (-4+1)^2$

$= (5)^2 + (-3)^2 = 25 + 9$

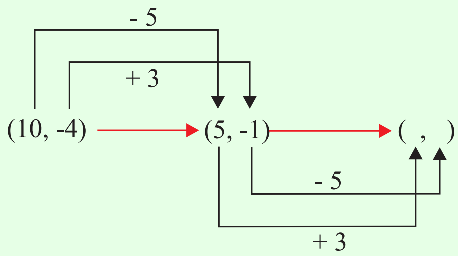
$= 34 \Rightarrow (10, -4) \in K$

3 (c) (iii)

FINDING A PARALLEL TANGENT TO A CIRCLE:
A tangent to a circle, T_1, has a parallel tangent, T_2, on the other side of the circle. The centre, o, is the midpoint of their points of contact, p and q. The slopes of the two tangents are the same.



You need to find the image of $(10, -4)$ under a central symmetry through the centre $(5, -1)$.



$$(10, -4) \to (5, -1) \to (0, 2)$$

Therefore, (0, 2) is the point of contact of tangent S with the circle K.