

## THE CIRCLE (Q 3, PAPER 2)

2005

- 3 (a) The circle  $C$  has equation  $x^2 + y^2 = 49$ .
- (i) Write down the centre and the radius of  $C$ .
  - (ii) Verify that the point  $(5, -5)$  lies outside the circle  $C$ .
- (b) The line  $y = 10 - 2x$  intersects the circle  $x^2 + y^2 = 40$  at the points  $a$  and  $b$ .
- (i) Find the coordinates of  $a$  and the co-ordinates of  $b$ .
  - (ii) Show the line, the circle and the points of intersection on a coordinate diagram.
- (c) The circle  $K$  has equation  $(x + 4)^2 + (y - 3)^2 = 36$ .
- (i) Write down the coordinates of the centre of  $K$ .
  - (ii) The point  $(2, 3)$  is one end-point of a diameter of  $K$ .  
Find the coordinates of the other end-point.
  - (iii) The point  $(-4, y)$  is on the circle  $K$ . Find the two values of  $y$ .

### SOLUTION

3 (a) (i)

Centre  $(0, 0)$ , radius  $r = \sqrt{49} = 7$

Circle  $C$  with centre  $(0, 0)$ , radius  $r$ .

$$x^2 + y^2 = r^2$$

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3 (a) (ii)

**IS A POINT ON A CIRCLE, INSIDE A CIRCLE OR OUTSIDE A CIRCLE?**

Substitute the point into the circle.

**On the circle:** Both sides are equal.

**Inside the circle:** The left hand side is less than the right hand side.

**Outside the circle:** The left hand side is greater than the right hand side.

To show that  $(5, -5)$  lies outside  $C$ :

$$C : x^2 + y^2 = 49$$

$$\Rightarrow (5)^2 + (-5)^2 = 25 + 25$$

$$= 50 > 49 \Rightarrow (5, -5) \text{ lies outside } C.$$

### 3 (b) (i)

#### STEPS

1. Isolate  $x$  or  $y$  using equation of the line.
2. Substitute into the equation of the circle and solve the resulting quadratic.

1.  $L: y = 10 - 2x$

2.  $C: x^2 + y^2 = 40$

$$\Rightarrow x^2 + (10 - 2x)^2 = 40$$

$$\Rightarrow x^2 + 100 - 40x + 4x^2 = 40$$

$$\Rightarrow 5x^2 - 40x + 60 = 0$$

$$\Rightarrow x^2 - 8x + 12 = 0$$

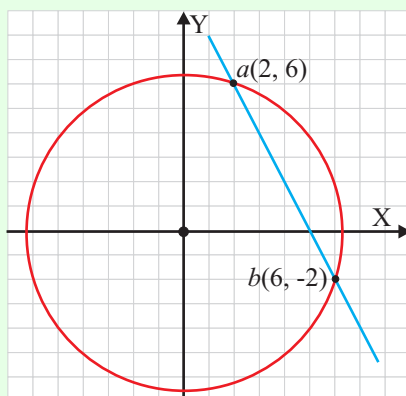
$$\Rightarrow (x - 2)(x - 6) = 0$$

$$\therefore x = 2, 6$$

$$\therefore y = 6, -2$$

Points of intersection:  $a(2, 6)$ ,  $b(6, -2)$

### 3 (b) (ii)



### 3 (c) (i)

Circle  $C$  with centre  $(h, k)$ , radius  $r$ .

$$(x - h)^2 + (y - k)^2 = r^2 \quad \text{..... } \textcircled{2}$$

**To get the centre:** Change the sign of the number inside each bracket.

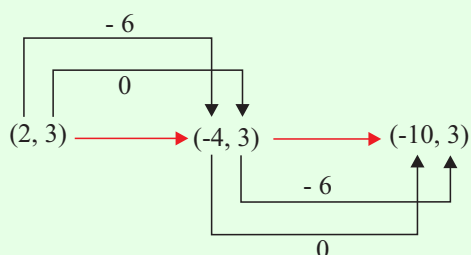
**To get the radius:** Take the square root of the number on the right.

$$K: (x + 4)^2 + (y - 3)^2 = 36$$

$$\text{Centre } (-4, 3), r = \sqrt{36} = 6$$

### 3 (c) (ii)

Find the image of  $(2, 3)$  by a central symmetry through the centre  $(-4, 3)$ .



$$(2, 3) \rightarrow (-4, 3) \rightarrow (-10, 3)$$

**3 (c) (iii)**

As  $(-4, y)$  is on the circle  $K$ , you can substitute it into the equation of  $K$ .

$$(-4, y) \in (x+4)^2 + (y-3)^2 = 36$$

$$\Rightarrow (-4+4)^2 + (y-3)^2 = 36$$

$$\Rightarrow 0 + (y-3)^2 = 36$$

$$\Rightarrow (y-3)^2 = 36$$

$$\Rightarrow (y-3) = \pm 6$$

$$\therefore y = -3, 9$$