THE CIRCLE (Q 3, PAPER 2)

2004

3 (a) The circle C has equation $x^2 + y^2 = 36$.

(i) Write down the radius of C.

(ii) The radius of another circle is twice the radius of C.
The centre of this circle is (0, 0). Write down its equation.

(b) A circle has equation $x^2 + y^2 = 13$.

The points $a(2, -3)$, $b(-2, 3)$ and $c(3, 2)$ are on the circle.

(i) Verify that $[ab]$ is a diameter of the circle.

(ii) Verify that $\angle acb$ is a right angle.

(c) K is a circle with centre $(-2, 1)$. It passes through the point $(-3, 4)$.

(i) Find the equation of K.

(ii) The point $(t, 2t)$ is on the circle K.
Find the two possible values of t.

SOLUTION

3 (a) (i)

$x^2 + y^2 = 36 \Rightarrow r = \sqrt{36} = 6$

Circle C with centre (0, 0), radius r.

$\boxed{x^2 + y^2 = r^2}$ 1

3 (a) (ii)

New circle: centre (0, 0), $r = 12$

$x^2 + y^2 = 12^2 \Rightarrow x^2 + y^2 = 144$

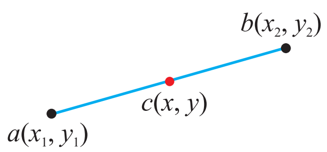
3 (b) (i)

TO SHOW A LINE SEGMENT IS A DIAMETER OF A CIRCLE:
The midpoint of a diameter is the centre of a circle.

The centre of the circle $x^2 + y^2 = 13$ is (0, 0).

The formula for the midpoint, c, of the line segment $[ab]$ is:

$\boxed{\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$ 2



REMEMBER THE MIDPOINT FORMULA AS: $\text{Midpoint} = \left(\frac{\text{Add the } x\text{'s}}{2}, \frac{\text{Add the } y\text{'s}}{2}\right)$

$$\begin{array}{cccc} a(2, & -3) & b(-2, & 3) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

Midpoint of $[ab] = \left(\frac{2-2}{2}, \frac{-3+3}{2}\right) = \left(\frac{0}{2}, \frac{0}{2}\right) = (0, 0)$

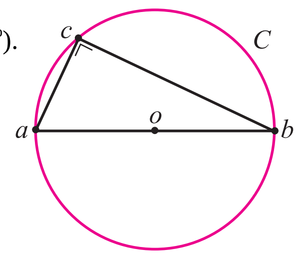
The midpoint of $[ab]$ is the centre of the circle. Therefore, $[ab]$ is a diameter of the circle.

3 (b) (ii)

The angle in a semi-circle at c is a right-angle (90°).
To prove this you need to show that ac is perpendicular to bc.

$$\boxed{\text{Slope of } ac \times \text{Slope of } bc = -1 \Rightarrow ac \perp bc}$$

This means that $[ab]$ is a diameter of a circle.



You can do this 3 ways:

1. Find the slope of ac and bc and show they are perpendicular.
2. Find the lengths of the 3 sides and apply Pythagoras' theorem.
3. Show c is on the circle. Any angle standing on the diameter is a right angle. This is my favourite and by far the quickest.

$$c(3, 2) \in x^2 + y^2 = 13?$$

$$(3)^2 + (2)^2 = 9 + 4$$

$$= 13 \Rightarrow x^2 + y^2 = 13$$

$$\therefore \angle acb \text{ is a right-angle.}$$

3 (c) (i)

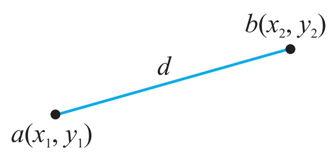
To form the equation of K you need its centre (given) and its radius (you can find the radius by working out the distance between the centre and the point on the circle).

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} \text{ ....} \quad 1$$

The distance between a and b is written as $|ab|$.

Remember the distance formula as:

$$d = \sqrt{(\text{Difference in } x\text{'s})^2 + (\text{Difference in } y\text{'s})^2}$$



$$\boxed{\begin{array}{cc} (-2, 1) & (-3, 4) \\ \downarrow \ \downarrow & \downarrow \ \downarrow \\ x_1 \ y_1 & x_2 \ y_2 \end{array}}$$

$$r = \sqrt{(-3-(-2))^2 + (4-1)^2}$$

$$\Rightarrow r = \sqrt{(-3+2)^2 + (4-1)^2}$$

$$\Rightarrow r = \sqrt{(-1)^2 + (3)^2} = \sqrt{1+9}$$

$$\therefore r = \sqrt{10}$$

Circle C with centre (h, k), radius r.

$$\boxed{(x-h)^2 + (y-k)^2 = r^2} \text{ ....} \quad 2$$

To get the centre: Change the sign of the number inside each bracket.
To get the radius: Take the square root of the number on the right.

Centre $(-2, 1)$, $r = \sqrt{10}$

$$K: (x+2)^2 + (y-1)^2 = (\sqrt{10})^2$$

$$\therefore (x+2)^2 + (y-1)^2 = 10$$

3 (c) (ii)

It the point (t, $2t$) lies on K, then you can substitute into K.

$(t,\ 2t) \in (x+2)^2+(y-1)^2=10$

$\Rightarrow (t+2)^2+(2t-1)^2=10$

$\Rightarrow t^2+4t+4+4t^2-4t+1=10$

$\Rightarrow 5t^2-5=0 \Rightarrow 5t^2=5$

$\Rightarrow t^2=1$

$\therefore t=\sqrt{1}=\pm 1$