THE CIRCLE (Q 3, PAPER 2)

2003

3 (a) The circle C has equation $x^2 + y^2 = 25$.

(i) Verify that the point $(-4, 3)$ is on the circle C.

(ii) Write down the coordinates of a point that lies outside C and give a reason for your answer.

(b) The line $x - 2y + 5 = 0$ intersects the circle $x^2 + y^2 = 10$ at the points a and b.

(i) Find the co-ordinates of a and the co-ordinates of b.

(ii) Draw a coordinate diagram on graph paper, showing the line, the circle and the points of intersection.

(c) The circle K has equation $(x+2)^2 + (y-3)^2 = 25$.

p and q are the endpoints of a diameter of K and pq is horizontal.

(i) Find the co-ordinates of p and the co-ordinates of q.

(ii) Hence, or otherwise, write down the equations of the two vertical tangents to K.

(iii) Another circle also has these two vertical lines as tangents.
The centre of this circle is on the x-axis.
Find the equation of this circle.

SOLUTION

3 (a) (i)

> **IS A POINT ON A CIRCLE, INSIDE A CIRCLE OR OUTSIDE A CIRCLE?**
> Substitute the point into the circle.
> **On the circle**: Both sides are equal.
> **Inside the circle**: The left hand side is less than the right hand side.
> **Outside the circle**: The left hand side is greater than the right hand side.

$(-4, 3) \in x^2 + y^2 = 25$?

$(-4)^2 + (3)^2 = 16 + 9$

$= 25 \Rightarrow (-4, 3) \in x^2 + y^2 = 25$

3 (a) (ii)

You need to pick a value of x and a value of y such that when you put it into the equation of the circle the left hand side is greater than 25.

$(4, 5)$ is such a number because $(4)^2 + (5)^2 = 16 + 25 = 41 > 25$.

3 (b) (i)

STEPS
1. Isolate x or y using equation of the line.
2. Substitute into the equation of the circle and solve the resulting quadratic.

1. $L: x-2y+5=0 \Rightarrow x=2y-5$

2. $C: x^2+y^2=10$

$\Rightarrow (2y-5)^2+y^2=10$

$\Rightarrow 4y^2-20y+25+y^2=10$

$\Rightarrow 5y^2-20y+15=0$

$\Rightarrow y^2-4y+3=0$

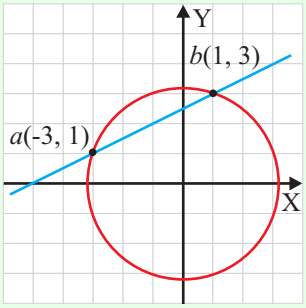
$\Rightarrow (y-1)(y-3)=0$

$\therefore y=1,\ 3$

$\therefore x=-3,\ 1$

Points of intersection: $a(-3,\ 1)$, $b(1,\ 3)$

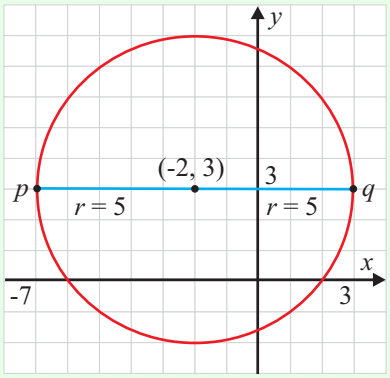
3 (b) (ii)



3 (c) (i)

Find the centre and radius of K. Draw in a horizontal diameter and work out the end points of the diameter by inspection.

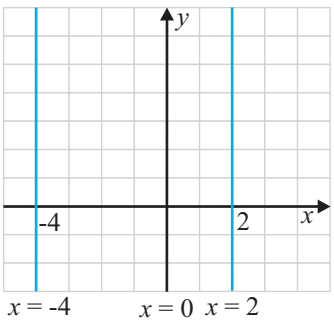
You can see from the diagram that the coordinates of the end points of the diameter are: $p(-7,\ 3)$, $q(3,\ 3)$



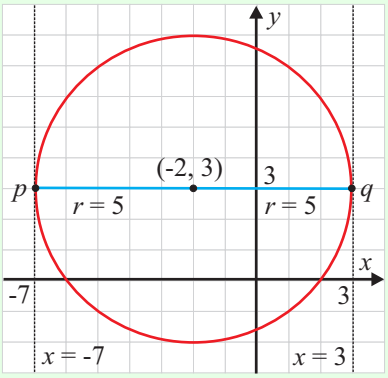
3 (c) (ii)

VERTICAL LINES

Vertical lines have equations where x = constant. In particular, the y-axis has the equation $x = 0$.



You can see from the diagram that the equations of the two vertical tangents are: $x = -7,\ x = 3$



3 (c) (iii)

The new circle has a centre $(-2,\ 0)$ and a radius 5.

New circle: $(x+2)^2 + (y-0)^2 = 5^2$

$$\therefore (x+2)^2 + y^2 = 25$$