THE CIRCLE (Q 3, PAPER 2)

2002

3 (a) Write down the coordinates of any three points that lie on the circle with equation $x^2 + y^2 = 100$.

(b) The circle C has equation $(x-2)^2 + (y+1)^2 = 8$.

(i) Find the coordinates of the two points at which C cuts the y-axis.

(ii) Find the equation of the tangent to C at the point $(4, 1)$.

(c) $a(-5, 1)$, $b(3, 7)$ and $c(9, -1)$ are three points.

(i) Show that the triangle abc is right-angled.

(ii) Hence, find the centre of the circle that passes through a, b and c and write down the equation of the circle.

SOLUTION

3 (a)

IS A POINT ON A CIRCLE, INSIDE A CIRCLE OR OUTSIDE A CIRCLE?
Substitute the point into the circle.
On the circle: Both sides are equal.
Inside the circle: The left hand side is less than the right hand side.
Outside the circle: The left hand side is greater than the right hand side.

You need to pick values of x and y which when you put them into the equation you get 100.

(6, 8) is a point on the circle because $(6)^2 + (8)^2 = 36 + 64 = 100$.

(8, 6) is a point on the circle because $(8)^2 + (6)^2 = 64 + 36 = 100$.

(10, 0) is a point on the circle because $(10)^2 + (0)^2 = 100 + 0 = 100$.

3 (b) (i)

TO FIND OUT WHERE THE CIRCLE, C, CROSSES THE x-AXIS:
Set $y = 0$ in the circle equation.
TO FIND OUT WHERE THE CIRCLE, C, CROSSES THE y-AXIS:
Set $x = 0$ in the circle equation.

$C: (x-2)^2 + (y+1)^2 = 8$

$x = 0 \Rightarrow (0-2)^2 + (y+1)^2 = 8$

$\Rightarrow 4 + (y+1)^2 = 8$

$\Rightarrow (y+1)^2 = 4$

$\Rightarrow (y+1) = \pm 2$

$\therefore y = -3, 1$

y-intercepts: $(0, -3), (0, 1)$

3 (b) (ii)

FINDING THE EQUATION OF A TANGENT, *T*, TO A CIRCLE:

STEPS
1. Find the slope of the line from the centre to the point of contact.
2. Find the slope of the tangent (it is perpendicular to the radius).
3. Find the equation of *T*.

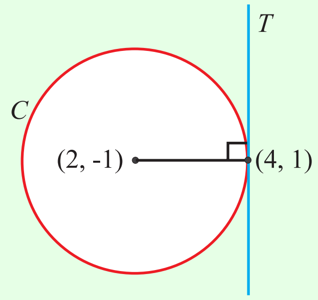
Circle C with centre (h, k), radius r.

$$\boxed{(x-h)^2+(y-k)^2=r^2} \quad \ldots\ldots\ \textbf{2}$$

To get the centre: Change the sign of the number inside each bracket.
To get the radius: Take the square root of the number on the right.

$C: (x-2)^2+(y+1)^2=8$

Centre $(2, -1)$



1. Find the slope of the line joining $(2, -1)$ to $(4, 1)$.

$$\boxed{m=\frac{y_2-y_1}{x_2-x_1}} \quad \ldots\ldots\ \textbf{3}$$

REMEMBER IT AS:

$$\text{Slope } m=\frac{\text{Difference in } y\text{'s}}{\text{Difference in } x\text{'s}}$$

$$\begin{array}{cccc} (2, & -1) & (4, & 1) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

Slope: $m=\dfrac{1-(-1)}{4-2}=\dfrac{1+1}{4-2}=\dfrac{2}{2}=1$

2. Slope of Tangent T:

FINDING THE PERPENDICULAR SLOPE: Invert the slope and change its sign.

$m=-1$

3. Equation of T: Point $(x_1, y_1) = (4, 1)$, slope $m=-1$

The equation of a line is a formula satisfied by every point (x, y) on the line.

Equation of a line: $\boxed{y-y_1=m(x-x_1)}$ **4**

$y-y_1=m(x-x_1)$

$\Rightarrow y-1=-1(x-4)$

$\Rightarrow y-1=-x+4$

$\therefore x+y-5=0$

3 (c) (i)

Find the slope of all 3 sides and show that two of the sides are perpendicular.

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}} \ \ldots\ldots\ \textbf{3}$$

Remember it as:

Slope $m = \dfrac{\text{Difference in } y\text{'s}}{\text{Difference in } x\text{'s}}$

$$\begin{array}{cccc} a(-5, & 1) & b(3, & 7) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

Slope of ab: $m_1 = \dfrac{7-1}{3-(-5)} = \dfrac{6}{3+5} = \dfrac{6}{8} = \dfrac{3}{4}$

$$\begin{array}{cccc} a(-5, & 1) & c(9, & -1) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

Slope of ac: $m_2 = \dfrac{-1-1}{9-(-5)} = \dfrac{-2}{9+5} = \dfrac{-2}{14} = -\dfrac{1}{7}$

$$\begin{array}{cccc} b(3, & 7) & c(9, & -1) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

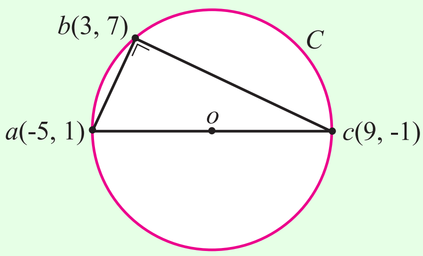
Slope of bc: $m_3 = \dfrac{-1-7}{9-3} = \dfrac{-8}{6} = -\dfrac{4}{3}$

Two lines are perpendicular if the product of their slopes is -1.

$$m_1 \times m_3 = (\tfrac{3}{4})(-\tfrac{4}{3}) = -1 \Rightarrow ab \perp bc$$

Therefore, the triangle abc is right-angled with the right angle at b.

3 (c) (ii)

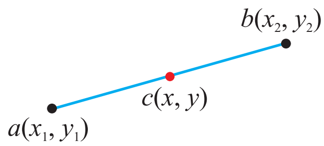


The angle in a semi-circle at b is a right-angle (90°). This means that $[ac]$ is a diameter of a circle.

The centre o is the midpoint of ac.

The formula for the midpoint, c, of the line segment $[ab]$ is:

$$\boxed{\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)} \ \ldots\ldots\ \textbf{2}$$



Remember the midpoint formula as: Midpoint $= \left(\dfrac{\text{Add the } x\text{'s}}{2}, \dfrac{\text{Add the } y\text{'s}}{2}\right)$

$$\begin{array}{cccc} a(-5, & 1) & c(9, & -1) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

Midpoint $o = \left(\dfrac{-5+9}{2}, \dfrac{1-1}{2}\right) = \left(\dfrac{4}{2}, \dfrac{0}{2}\right) = (2, 0)$

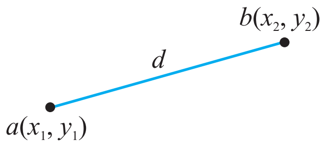
The radius is the distance from the centre o to any vertex on the triangle, say $b(3, 7)$.

$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$ 1

The distance between a and b is written as $|ab|$.

Remember the distance formula as:

$d = \sqrt{(\text{Difference in } x\text{'s})^2 + (\text{Difference in } y\text{'s})^2}$



$$\begin{matrix} o(2, 0) & b(3, 7) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ y_2 \end{matrix}$$

$r = |ob| = \sqrt{(3-2)^2 + (7-0)^2}$

$\Rightarrow |ob| = \sqrt{(1)^2 + (7)^2}$

$\Rightarrow |ob| = \sqrt{1 + 49}$

$\therefore r = \sqrt{50}$

Equation of the circle: centre $(h, k) = (2, 0)$, $r = \sqrt{50}$

Circle C with centre (h, k), radius r.

$\boxed{(x-h)^2 + (y-k)^2 = r^2}$ 2

To get the centre: Change the sign of the number inside each bracket.
To get the radius: Take the square root of the number on the right.

$(x-h)^2 + (y-k)^2 = r^2$

$\Rightarrow (x-2)^2 + (y-0)^2 = (\sqrt{50})^2$

$\therefore (x-2)^2 + y^2 = 50$