THE CIRCLE (Q 3, PAPER 2)

2001

3 (a) The circle S has equation $(x-3)^2+(y-4)^2=25$.

(i) Write down the centre and the radius of S.

(ii) The point $(k, 0)$ lies on S. Find the two real values of k.

(b) Prove that the line $x-3y=10$ is a tangent to the circle with equation $x^2+y^2=10$ and find the coordinates of the point of contact.

(c) C is a circle with centre $(0, 0)$. It passes through the point $(1, -5)$.

(i) Write down the equation of C.

(ii) The point (p, p) lies inside C where $p \in \mathbf{Z}$.
Find all the possible values of p.

SOLUTION

3 (a) (i)

Circle C with centre (h, k), radius r.

$\boxed{(x-h)^2+(y-k)^2=r^2}$ 2

To get the centre: Change the sign of the number inside each bracket.
To get the radius: Take the square root of the number on the right.

$S:(x-3)^2+(y-4)^2=25$

Centre $(3, 4)$, $r=\sqrt{25}=5$

3 (a) (ii)

IS A POINT ON A CIRCLE, INSIDE A CIRCLE OR OUTSIDE A CIRCLE?
Substitute the point into the circle.
On the circle: Both sides are equal.
Inside the circle: The left hand side is less than the right hand side.
Outside the circle: The left hand side is greater than the right hand side.

$(k, 0)\in S \Rightarrow (k-3)^2+(0-4)^2=25$

$\Rightarrow (k-3)^2+(-4)^2=25$

$\Rightarrow k^2-6k+9+16=25$

$\Rightarrow k^2-6k=0$

$\Rightarrow k(k-6)=0$

$\therefore k=0, 6$

3 (b)

Proof that a line is a tangent to a circle: When you solve the quadratic only one answer is obtained, i.e. one point of contact.

Steps
1. Isolate x or y using equation of the line.
2. Substitute into the equation of the circle and solve the resulting quadratic.

1. $L: x-3y=10 \Rightarrow x=3y+10$

2. $C: x^2+y^2=10$

$\Rightarrow (3y+10)^2+y^2=10$

$\Rightarrow 9y^2+60y+100+y^2=10$

$\Rightarrow 10y^2+60y+90=0$

$\Rightarrow y^2+6y+9=0$

$\Rightarrow (y+3)(y+3)=0$

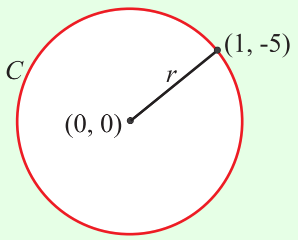
$\therefore y=-3$

$\therefore x=3y+10=3(-3)+10=-9+10=1$

Points of intersection: $(1, -3)$

As there is only one point of contact, the line is a tangent to the circle.

3 (c) (i)

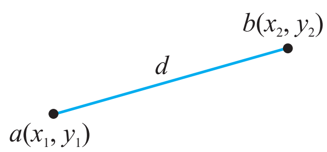


Circle C with centre (0, 0), radius r.

$\boxed{x^2+y^2=r^2}$ **1**

Find the radius by finding the distance between the centre (0, 0) and the point on the circle $(1, -5)$.

$\boxed{d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}$ **1**



The distance between a and b is written as $|ab|$.

Remember the distance formula as:

$d=\sqrt{(\text{Difference in } x\text{'s})^2+(\text{Difference in } y\text{'s})^2}$

$$\begin{array}{cc} (0,\ 0) & (1,\ -5) \\ \downarrow\ \downarrow & \downarrow\quad \downarrow \\ x_1\ y_1 & x_2\quad y_2 \end{array}$$

$r=\sqrt{(1-0)^2+(-5-0)^2}$

$\Rightarrow r=\sqrt{1^2+(-5)^2}=\sqrt{1+25}$

$\therefore r=\sqrt{26}$

Equation of C: Centre (0, 0), $r=\sqrt{26}$

$C: x^2+y^2=(\sqrt{26})^2 \Rightarrow x^2+y^2=26$

3 (c) (ii)

Do this by inspection. *p* is an integer which is a whole number (positive and negative.)

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$p=0:(0,\ 0)\Rightarrow(0)^2+(0)^2=0<26\Rightarrow(0,\ 0)$ is inside the circle.

$p=1:(1,\ 1)\Rightarrow(1)^2+(1)^2=1+1=2<26\Rightarrow(1,\ 1)$ is inside the circle.

$p=-1:(-1,\ -1)\Rightarrow(-1)^2+(-1)^2=1+1=2<26\Rightarrow(-1,\ -1)$ is inside the circle.

$p=2:(2,\ 2)\Rightarrow(2)^2+(2)^2=4+4=8<26\Rightarrow(2,\ 2)$ is inside the circle.

$p=-2:(-2,\ -2)\Rightarrow(-2)^2+(-2)^2=4+4=8<26\Rightarrow(-2,\ -2)$ is inside the circle.

$p=3:(3,\ 3)\Rightarrow(3)^2+(3)^2=9+9=18<26\Rightarrow(3,\ 3)$ is inside the circle.

$p=-3:(-3,\ -3)\Rightarrow(-3)^2+(-3)^2=9+9=18<26\Rightarrow(-3,\ -3)$ is inside the circle.

$p=4:(4,\ 4)\Rightarrow(4)^2+(4)^2=16+16=32>26\Rightarrow(4,\ 4)$ is outside the circle.

$p=-4:(-4,\ -4)\Rightarrow(-4)^2+(-4)^2=16+16=32>26\Rightarrow(-4,\ -4)$ is outside the circle.

All whole numbers of *p* between -3 and 3 give rise to points inside the circle.

ANOTHER WAY:
Find out the values of *p* for which (*p*, *p*) is on the circle.

$(p,\ p)\in x^2+y^2=26\Rightarrow(p)^2+(p)^2=26$

$\Rightarrow 2p^2=26$

$\Rightarrow p^2=13$

$\therefore p=\pm\sqrt{13}\approx\pm3.6$

Therefore, the point (*p*, *p*) is inside the circle for values of *p* between $-\sqrt{13}$ and $\sqrt{13}$.

Therefore, the point (*p*, *p*) is inside the circle for whole number values of *p* between -3 and 3.

ANS: $p=\{-3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3\}$