THE CIRCLE (Q 3, PAPER 2)

2000

3 (a) The circle C has equation $x^2 + y^2 = 16$.

(i) Write down the length of the radius of C.

(ii) Show, by calculation, that the point (3, 1) is inside the circle.

(b) (i) Find the slope of the tangent to the circle $x^2 + y^2 = 29$ at the point (2, 5).

(ii) Hence, find the equation of the tangent.

(c) (i) The end points of a diameter of a circle are (– 2, –3) and (– 4, 3). Find the equation of the circle.

(ii) The circle cuts the y-axis at the points a and b. Find $|ab|$.

(iii) c and d are points on the circle such that $abcd$ is a rectangle. Find the area of the rectangle $abcd$.

SOLUTION

3 (a) (i)

$C: x^2 + y^2 = 16$

$\Rightarrow r = \sqrt{16} = 4$

Circle C with centre (0, 0), radius r.

$\boxed{x^2 + y^2 = r^2}$ 1

3 (a) (ii)

IS A POINT ON A CIRCLE, INSIDE A CIRCLE OR OUTSIDE A CIRCLE?
Substitute the point into the circle.
On the circle: Both sides are equal.
Inside the circle: The left hand side is less than the right hand side.
Outside the circle: The left hand side is greater than the right hand side.

$(3)^2 + (1)^2 = 9 + 1$

$= 10 < 16 \Rightarrow$ (3, 1) is inside the circle.

3 (b) (i)

Finding the equation of a tangent, T, to a circle:

Steps
1. Find the slope of the line from the centre to the point of contact.
2. Find the slope of the tangent (it is perpendicular to the radius).
3. Find the equation of T.

Centre of $x^2 + y^2 = 29$: (0, 0)

Circle C with centre (0, 0), radius r.

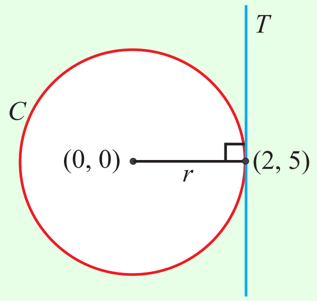
$\boxed{x^2 + y^2 = r^2}$ **1**

1. Find the slope of the line joining (0, 0) to (2, 5).

$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$ **3**

Remember it as:

Slope $m = \frac{\text{Difference in } y\text{'s}}{\text{Difference in } x\text{'s}}$



$$\begin{array}{cc} (0,\ 0) & (2,\ 5) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

Slope: $m = \frac{5-0}{2-0} = \frac{5}{2}$

2. The slope of the tangent is perpendicular to this slope.

Finding the perpendicular slope: Invert the slope and change its sign.

Slope of tangent: $m = -\frac{2}{5}$

3 (b) (ii)

3. Equation of T: Point $(x_1, y_1) = (2, 5)$, slope $m = -\frac{2}{5}$

The equation of a line is a formula satisfied by every point (x, y) on the line.

Equation of a line: $\boxed{y - y_1 = m(x - x_1)}$ **4**

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 5 = -\frac{2}{5}(x - 2)$

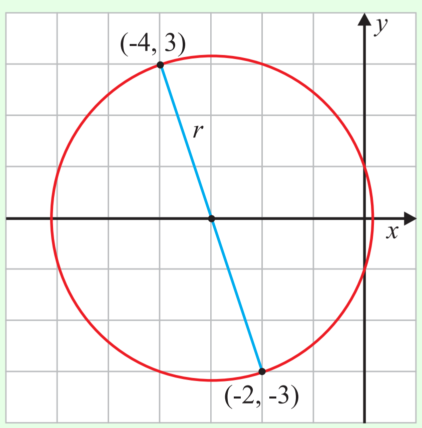
$\Rightarrow 5(y - 5) = -2(x - 2)$

$\Rightarrow 5y - 25 = -2x + 4$

$\therefore 2x + 5y - 29 = 0$

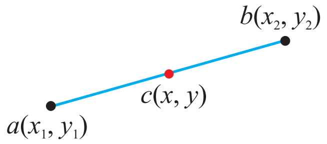
3 (c) (i)

To find the centre, find the midpoint of the end points of the diameter.



The formula for the midpoint, c, of the line segment $[ab]$ is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \quad \ldots\ldots\ \textbf{2}$$



Remember the midpoint formula as: $\text{Midpoint} = \left(\frac{\text{Add the } x\text{'s}}{2}, \frac{\text{Add the } y\text{'s}}{2} \right)$

$$\begin{array}{cccc} (-2, & -3) & (-4, & 3) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

$$\text{Midpoint} = \left(\frac{-2-4}{2}, \frac{-3+3}{2} \right) = \left(\frac{-6}{2}, \frac{0}{2} \right) = (-3, 0)$$

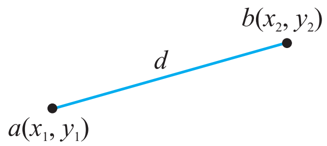
To find the centre, find the distance between the centre $(-3, 0)$ and either end point, say $(-2, -3)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \ldots\ldots\ \textbf{1}$$

The distance between a and b is written as $|ab|$.

Remember the distance formula as:

$$d = \sqrt{(\text{Difference in } x\text{'s})^2 + (\text{Difference in } y\text{'s})^2}$$



$$\begin{array}{cccc} (-3, & 0) & (-2, & -3) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

$$r = \sqrt{(-2-(-3))^2 + (-3-0)^2}$$

$$\Rightarrow r = \sqrt{(-2+3)^2 + (-3-0)^2}$$

$$\Rightarrow r = \sqrt{(1)^2 + (-3)^2} = \sqrt{1+9}$$

$$\therefore r = \sqrt{10}$$

Equation of circle: centre $(-3, 0)$, $r = \sqrt{10}$

Circle C with centre (h, k), radius r.

$$(x-h)^2 + (y-k)^2 = r^2 \quad \ldots\ldots\ \textbf{2}$$

To get the centre: Change the sign of the number inside each bracket.
To get the radius: Take the square root of the number on the right.

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\Rightarrow (x-(-3))^2 + (y-0)^2 = (\sqrt{10})^2$$

$$\Rightarrow (x+3)^2 + y^2 = 10$$

3 (c) (ii)

TO FIND OUT WHERE THE CIRCLE, *C*, CROSSES THE *x*-AXIS:
Set $y = 0$ in the circle equation.
TO FIND OUT WHERE THE CIRCLE, *C*, CROSSES THE *y*-AXIS:
Set $x = 0$ in the circle equation.

$x = 0 \Rightarrow (0+3)^2 + y^2 = 10$

$\Rightarrow (3)^2 + y^2 = 10$

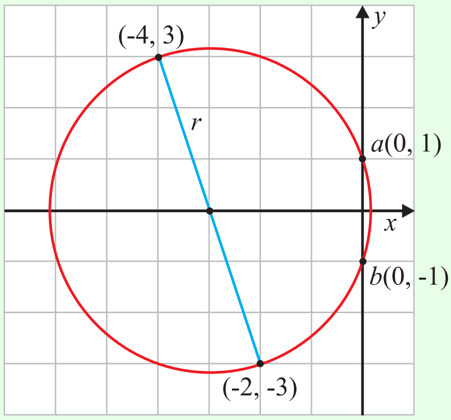
$\Rightarrow 9 + y^2 = 10$

$\Rightarrow y^2 = 1$

$\therefore y = \sqrt{1} = \pm 1$

$\therefore a(0, 1), b(0, -1)$ are the y-intercepts.

As you can see from the diagram the distance $|ab| = 2$.



3 (c) (iii)

Area of rectangle *abcd* = $6 \times 2 = 12$ square units