THE CIRCLE (Q 3, PAPER 2)

1999

3 (a) C is a circle with centre (0, 0) passing through the point (8, 6).
Find
(i) the radius length of C

(ii) the equation of C.

(b) The points $(-1, -1)$ and $(3, -3)$ are the end points of a diameter of a circle S.
(i) Find the coordinates of the centre of S.

(ii) Find the radius length of S.

(iii) Find the equation of S.

(c) A circle K has equation $x^2 + y^2 = 13$.
T is a tangent to K at $(-2, -3)$.
Find the equation of T.
Find the equation of the other tangent to K which is parallel to T.

SOLUTION

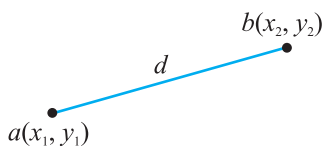
3 (a) (i)

$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$ 1

The distance between a and b is written as $|ab|$.

REMEMBER THE DISTANCE FORMULA AS:

$d = \sqrt{(\text{Difference in } x\text{'s})^2 + (\text{Difference in } y\text{'s})^2}$

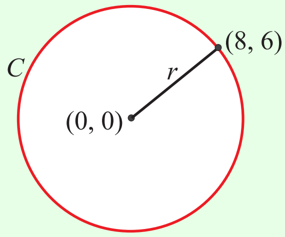


$$\begin{matrix} (0, & 0) & (8, & 6) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{matrix}$$

$r = \sqrt{(8-0)^2 + (6-0)^2}$

$\Rightarrow r = \sqrt{8^2 + 6^2} = \sqrt{64 + 36}$

$\therefore r = \sqrt{100} = 10$



3 (a) (ii)

Equation of C: centre (0, 0), r = 10

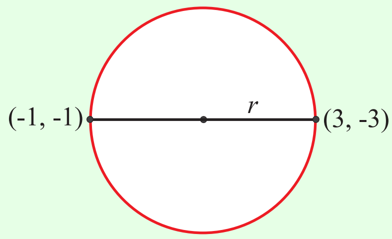
$C: x^2 + y^2 = 100$

Circle C with centre (0, 0), radius r.

$\boxed{x^2 + y^2 = r^2}$ 1

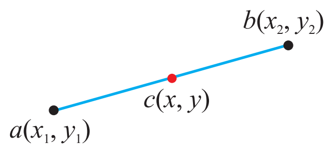
3 (b) (i)

The centre of the circle is the midpoint of the end points of the diameter.



The formula for the midpoint, c, of the line segment $[ab]$ is:

$$\boxed{\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)} \quad \ldots\ldots\ 2$$



REMEMBER THE MIDPOINT FORMULA AS: $\text{Midpoint} = \left(\frac{\text{Add the } x\text{'s}}{2}, \frac{\text{Add the } y\text{'s}}{2}\right)$

$$\begin{array}{cccc} (-1, & -1) & (3, & -3) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

$$\text{Midpoint} = \left(\frac{-1+3}{2}, \frac{-1-3}{2}\right) = \left(\frac{2}{2}, \frac{-4}{2}\right) = (1, -2)$$

3 (b) (ii)

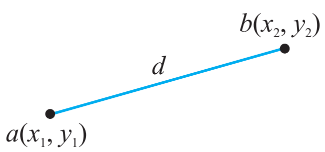
The radius is the distance from the centre $(1, -2)$ to either of the end points of the diameter, say $(-1, -1)$.

$$\boxed{d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}} \quad \ldots\ldots\ 1$$

The distance between a and b is written as $|ab|$.

REMEMBER THE DISTANCE FORMULA AS:

$$d = \sqrt{(\text{Difference in } x\text{'s})^2 + (\text{Difference in } y\text{'s})^2}$$



$$\begin{array}{cccc} (1, & -2) & (-1, & -1) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

$$r = \sqrt{(-1-1)^2+(-1-(-2))^2}$$

$$\Rightarrow r = \sqrt{(-1-1)^2+(-1+2)^2}$$

$$\Rightarrow r = \sqrt{(-2)^2+(1)^2} = \sqrt{4+1}$$

$$\therefore r = \sqrt{5}$$

3 (b) (iii)

Equation of S: centre $(h, k) = (1, -2)$, $r = \sqrt{5}$

Circle C with centre (h, k), radius r.

$$\boxed{(x-h)^2+(y-k)^2 = r^2} \quad \ldots\ldots\ 2$$

To get the centre: Change the sign of the number inside each bracket.
To get the radius: Take the square root of the number on the right.

$$S:(x-h)^2+(y-k)^2=r^2$$
$$\Rightarrow (x-1)^2+(y-(-2))^2=(\sqrt{5})^2$$
$$\therefore (x-1)^2+(y+2)^2=5$$

3 (c) **Finding the equation of a tangent, T, to a circle:**

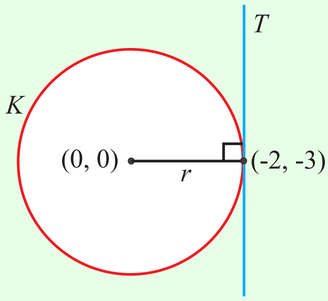
Steps
1. Find the slope of the line from the centre to the point of contact.
2. Find the slope of the tangent (it is perpendicular to the radius).
3. Find the equation of T.

Centre of $x^2+y^2=13$: $(0, 0)$

Circle C with centre (0, 0), radius r.

$\boxed{x^2+y^2=r^2}$ 1

1. Find the slope of the line joining (0, 0) to $(-2, -3)$.



$\boxed{m=\dfrac{y_2-y_1}{x_2-x_1}}$ 3

Remember it as:

$$\text{Slope } m=\frac{\text{Difference in } y\text{'s}}{\text{Difference in } x\text{'s}}$$

$$\begin{array}{cc} (0,\ 0) & (-2,\ -3) \\ \downarrow\ \downarrow & \downarrow\ \ \downarrow \\ x_1\ y_1 & x_2\ \ y_2 \end{array}$$

$$\text{Slope: } m=\frac{-3-0}{-2-0}=\frac{-3}{-2}=\frac{3}{2}$$

2. The slope of the tangent is perpendicular to this slope.

Finding the perpendicular slope: Invert the slope and change its sign.

Slope of tangent T: $m=-\frac{2}{3}$

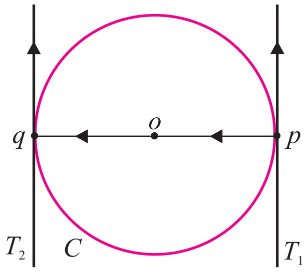
3. Equation of T: Point $(x_1, y_1)=(-2, -3)$, slope $m=-\frac{2}{3}$

The equation of a line is a formula satisfied by every point (x, y) on the line.

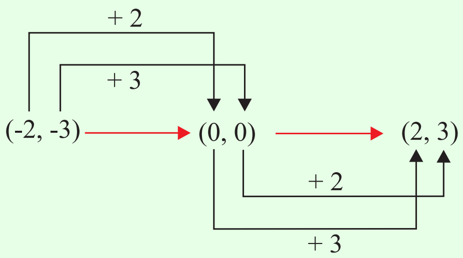
Equation of a line: $\boxed{y-y_1=m(x-x_1)}$ 4

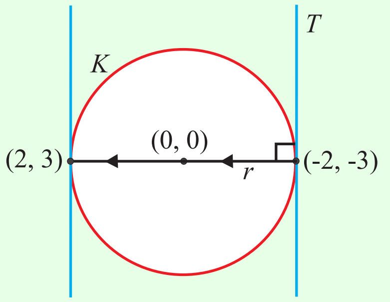
$$T: y-y_1=m(x-x_1)$$
$$\Rightarrow y-(-3)=-\tfrac{2}{3}(x-(-2))$$
$$\Rightarrow 3(y+3)=-2(x+2)$$
$$\Rightarrow 3y+9=-2x-4$$
$$\therefore 2x+3y+13=0$$

FINDING A PARALLEL TANGENT TO A CIRCLE:
A tangent to a circle, T_1, has a parallel tangent, T_2, on the other side of the circle. The centre, o, is the midpoint of their points of contact, p and q. The slopes of the two tangents are the same.



To find the point of contact of the parallel tangent, find the image of $(-2, -3)$ by a central symmetry through the centre (0, 0).





$(-2, -3) \rightarrow (0, 0) \rightarrow (2, 3)$

Equation of parallel tangent: Point $(x_1, y_1) = (2, 3)$, slope $m = -\frac{2}{3}$

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 3 = -\frac{2}{3}(x - 2)$

$\Rightarrow 3(y - 3) = -2(x - 2)$

$\Rightarrow 3y - 9 = -2x + 4$

$\therefore 2x + 3y - 13 = 0$