## The Circle (Q 3, Paper 2)

## 1998

3 (a) A circle $C$, with centre $(0,0)$, passes through the point $(4,-3)$.
(i) Find the length of the radius of $C$.
(ii) Show, by calculation, that the point $(6,-1)$ lies outside $C$.
(b) The equation of the circle $K$ is $(x-3)^{2}+(y+2)^{2}=29$.
(i) Write down the radius length and the coordinates of the centre of $K$.
(ii) Find the coordinates of the two points where $K$ intersects the $x$-axis.
(c) The line with equation $3 x-y+10=0$ is a tangent to the circle which has equation $x^{2}+y^{2}=10$.
(i) Find the coordinates of $a$, the point at which the line touches the circle.
(ii) The origin is the midpoint of [ab].

Find the equation of the tangent to the circle at $b$.

## Solution

3 (a) (i)

$$
d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}} \ldots \ldots .
$$

The distance between $a$ and $b$ is written as $|a b|$.
Remember the distance formula as:

$d=\sqrt{\left(\text { Difference in } x^{\prime} \mathrm{s}\right)^{2}+\left(\text { Difference in } y^{\prime} \mathrm{s}\right)^{2}}$

$$
\begin{array}{ccc}
\hline(0,0) & (4,-3) \\
\downarrow \downarrow & \downarrow & \downarrow \\
x_{1} y_{1} & x_{2} & y_{2}
\end{array} \quad \begin{aligned}
& r=\sqrt{(4-0)^{2}+(-3-0)^{2}} \\
&
\end{aligned}
$$



## 3 (a) (ii)

You can show that the distance $d$ from the centre $(0,0)$ to $(6,-1)$ is greater than the radius.

$$
\begin{array}{ll}
\begin{array}{ccc}
\begin{array}{ccc}
(0,0) & (6,-1) \\
\downarrow \downarrow & \downarrow & \downarrow \\
x_{1} y_{1} & x_{2} & y_{2}
\end{array} & & d=\sqrt{(6-0)^{2}+(-1-0)^{2}} \\
& \Rightarrow d=\sqrt{(6)^{2}+(-1)^{2}}=\sqrt{36+1} \\
d>r \text { as } \sqrt{37}>\sqrt{25} . & \therefore d=\sqrt{37}
\end{array} \\
\end{array}
$$

3 (b) (i)
Circle $C$ with centre $(h, k)$, radius $r$.

$$
\begin{equation*}
(x-h)^{2}+(y-k)^{2}=r^{2} \tag{2}
\end{equation*}
$$

To get the centre: Change the sign of the number inside each bracket. To get the radius: Take the square root of the number on the right.
$K:(x-3)^{2}+(y+2)^{2}=29$
Centre (3, - 2 ), $r=\sqrt{29}$
3 (b) (ii)
To find out where the circle, $C$, crosses the $x$-axis: Set $y=0$ in the circle equation.
To find out where the circle, $C$, crosses the $y$-axis:
Set $x=0$ in the circle equation.
$y=0 \Rightarrow(x-3)^{2}+(0+2)^{2}=29$
$\Rightarrow(x-3)^{2}+(2)^{2}=29$
$\Rightarrow(x-3)^{2}+4=29$
$\Rightarrow(x-3)^{2}=25$
$\Rightarrow(x-3)= \pm 5$
$\therefore x=-2,8$
$\therefore(-2,0),(8,0)$ are the $x$-intercepts.
3 (c) (i) Finding the point of contact between a tangent and a circle:

## Steps

1. Isolate $x$ or $y$ using equation of the line.
2. Substitute into the equation of the circle and solve the resulting quadratic.

Proof that a line is a tangent to a circle: When you solve the quadratic only one answer is obtained, i.e. one point of contact.

1. $3 x-y+10=0 \Rightarrow y=(3 x+10)$
2. $x^{2}+y^{2}=10$
$\Rightarrow x^{2}+(3 x+10)^{2}=10$
$\Rightarrow x^{2}+9 x^{2}+60 x+100=10$
$\Rightarrow 10 x^{2}+60 x+90=0$
$\Rightarrow x^{2}+6 x+9=0$
$\Rightarrow(x+3)(x+3)=0$
$\therefore x=-3 \Rightarrow y=3(-3)+10=-9+10=1$
$\therefore a(-3,1)$ is the point of contact.

## 3 (c) (ii)

Finding a parallel tangent to a circle:
A tangent to a circle, $T_{1}$, has a parallel tangent, $T_{2}$, on the other side of the circle. The centre, $o$, is the midpoint of their points of contact, $p$ and $q$. The slopes of the two tangents are the same.


To find the point of contact of the parallel tangent, find the image of $(-3,1)$ by a central symmetry through the centre ( 0,0 ).

$a(-3,1) \rightarrow(0,0) \rightarrow b(3,-1)$
Find the slope of the tangent with equation $3 x-y+10=0$.

## General form of a straight line

Every straight line can be written in the form: $a x+b y+c=0$. You can read off the slope of a straight line from its equation.

$$
\text { Slope: } m=-\left(\frac{a}{b}\right) \ldots \ldots
$$

Remember it as: Slope $m=-\left(\frac{\text { Number in front of } x}{\text { Number in front of } y}\right)$

Therefore, the slope of the tangent: $m=+\frac{3}{1}=3$

Parallel lines have the same slope.

Equation of parallel tangent: Point $\left(x_{1}, y_{1}\right)=b(3,-1), m=3$
The equation of a line is a formula satisfied by every point $(x, y)$ on the line.
Equation of a line:
$y-y_{1}=m\left(x-x_{1}\right)$
4
$y-y_{1}=m\left(x-x_{1}\right)$
$\Rightarrow y-(-1)=3(x-3)$
$\Rightarrow y+1=3 x-9$
$\therefore 3 x-y-10=0$

