THE CIRCLE (Q 3, PAPER 2)

1997

3 (a) The equation of a circle is $x^2 + y^2 = 49$.
Write down
(i) its radius length

(ii) the coordinates of the points where it intersects the x-axis.

(b) Prove that the line $x - 2y + 10 = 0$ is a tangent to the circle whose equation is $x^2 + y^2 = 20$.

(c) C is the circle with centre $(-1, 2)$ and radius 5.
Write down the equation of C.
The circle K has equation $(x-8)^2 + (y-14)^2 = 100$.
Prove that the point $p(2, 6)$ is on C and on K.
Show that p lies on the line which joins the centres of the two circles.

SOLUTION

3 (a) (i)

Circle C with centre (0, 0), radius r.

$\boxed{x^2 + y^2 = r^2}$ 1

$x^2 + y^2 = 49$

$\Rightarrow r = \sqrt{49} = 7$

3 (a) (ii)

TO FIND OUT WHERE THE CIRCLE, C, CROSSES THE x-AXIS:
Set $y = 0$ in the circle equation.
TO FIND OUT WHERE THE CIRCLE, C, CROSSES THE y-AXIS:
Set $x = 0$ in the circle equation.

$y = 0 \Rightarrow x^2 + (0)^2 = 49$

$\Rightarrow x^2 = 49$

$\therefore x = \sqrt{49} = \pm 7$

$\therefore (-7, 0), (7, 0)$ are the x-intercepts.

3 (b)

FINDING THE POINT OF CONTACT BETWEEN A TANGENT AND A CIRCLE:

STEPS
1. Isolate x or y using equation of the line.
2. Substitute into the equation of the circle and solve the resulting quadratic.

PROOF THAT A LINE IS A TANGENT TO A CIRCLE: When you solve the quadratic only one answer is obtained, i.e. one point of contact.

1. $x-2y+10=0 \Rightarrow x=2y-10$

2. $x^2+y^2=20$

$\Rightarrow (2y-10)^2+y^2=20$

$\Rightarrow 4y^2-40y+100+y^2=20$

$\Rightarrow 5y^2-40y+80=0$

$\Rightarrow y^2-8y+16=0$

$\Rightarrow (y-4)(y-4)=0$

$\therefore y=4$

As the quadratic has only one solution, the line is a tangent to the circle.

3 (c)

Equation of C: centre $(h,\ k)=(-1,\ 2)$, $r=5$

$(x-h)^2+(y-k)^2=r^2$

$\Rightarrow (x-(-1))^2+(y-2)^2=(5)^2$

$\therefore (x+1)^2+(y-2)^2=25$

IS A POINT ON A CIRCLE, INSIDE A CIRCLE OR OUTSIDE A CIRCLE?
Substitute the point into the circle.
On the circle: Both sides are equal.
Inside the circle: The left hand side is less than the right hand side.
Outside the circle: The left hand side is greater than the right hand side.

$C:(x+1)^2+(y-2)^2=25$

$p(2,\ 6)\in C$?

$(2+1)^2+(6-2)^2=(3)^2+(4)^2$

$=9+16=25 \Rightarrow p(2,\ 6)\in C$

$K:(x-8)^2+(y-14)^2=100$

$p(2,\ 6)\in K$?

$(2-8)^2+(6-14)^2=(-6)^2+(8)^2$

$=36+64=100 \Rightarrow p(2,\ 6)\in K$

Write down the centres of C and K.

Circle C with centre (h, k), radius r.

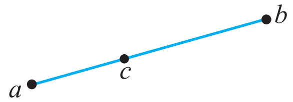
$\boxed{(x-h)^2+(y-k)^2=r^2}$ 2

To get the centre: Change the sign of the number inside each bracket.
To get the radius: Take the square root of the number on the right.

$C: (x+1)^2+(y-2)^2=25 \Rightarrow$ centre $p_1(-1, 2)$

$K: (x-8)^2+(y-14)^2=100 \Rightarrow$ centre $p_2(8, 14)$

COLLINEAR POINTS: Three points are collinear if the slope of any two points equals the slope of any other two points.
Ex. a, b and c are collinear if you can show that:
Slope of ac = Slope of cb



To show all three points are on the same line (collinear), show that the slope of pp_1 equals the slope of pp_2.

$\boxed{m=\dfrac{y_2-y_1}{x_2-x_1}}$ 3

REMEMBER IT AS:

Slope $m=\dfrac{\text{Difference in } y\text{'s}}{\text{Difference in } x\text{'s}}$

$$\begin{array}{cccc} p(2, & 6) & p_1(-1, & 2) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

Slope of pp_1: $m_1=\dfrac{2-6}{-1-2}=\dfrac{-4}{-3}=\dfrac{4}{3}$

$$\begin{array}{cccc} p(2, & 6) & p_2(8, & 14) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

Slope of pp_2: $m_2=\dfrac{14-6}{8-2}=\dfrac{8}{6}=\dfrac{4}{3}$

Therefore, p lies on the line which joins the centres of the two circles.