AREA & VOLUME (Q 1, PAPER 2)

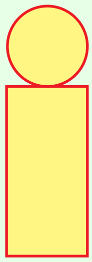
LESSON NO. 3: SURFACE AREA & VOLUME OF REGULAR SHAPES

2007

1 (c) A team trophy for the winners of a football match is in the shape of a sphere supported on a cylindrical base, as shown. The diameter of the sphere and of the cylinder is 21 cm.

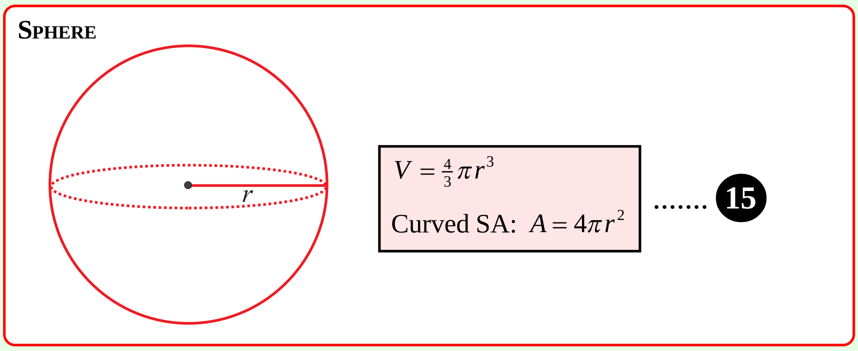
(i) Find the volume of the sphere, in terms of π.

(ii) The volume of the trophy is 6174π cm^3. Find the height of the cylinder.



SOLUTION

1 (c) (i)

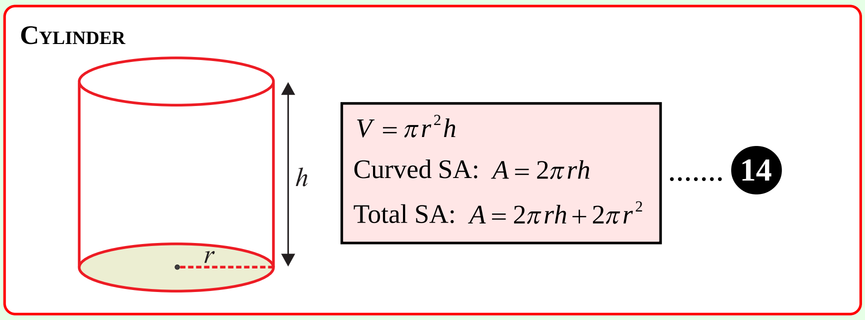


$r = \frac{21}{2}$ cm

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(\frac{21}{2})^3$

$\therefore V = 1543.5\pi$ cm^3

1 (c) (ii)



Volume of trophy = Volume of sphere + Volume of cylinder

$\therefore$ Volume of sphere = Volume of trophy – Volume of cylinder

$= 6174\pi - 1543.5\pi = 4630.5\pi$ cm^3

CYLINDER:

$V = 4630.5\pi$ cm^3

$r = \frac{21}{2}$ cm

$$V = \pi r^2 h \Rightarrow h = \frac{V}{\pi r^2}$$

$$\therefore h = \frac{4630.5\pi}{\pi(\frac{21}{2})^2} = 42 \text{ cm}$$

2006

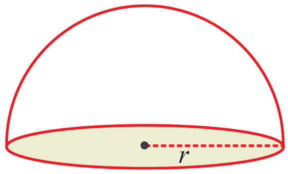
1 (c) (i) The volume of a hemisphere is 486π cm^3.
Find the radius of the hemisphere.

(ii) Find the volume of the smallest rectangular box that the hemisphere will fit into.

SOLUTION

1 (c) (i)

HEMISPHERE



$V = \frac{2}{3}\pi r^3$

Curved SA: $A = 2\pi r^2$

Total SA: $A = 3\pi r^2$ 16

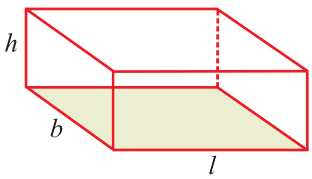
$V = 486\pi$

$V = \frac{2}{3}\pi r^3 \Rightarrow 486\pi = \frac{2}{3}\pi r^3$

$\Rightarrow r^3 = \frac{3 \times 486}{2} = 729$

$\therefore r = \sqrt[3]{729} = 9$ cm

RECTANGULAR SOLID



l: Length
b: Breadth
h: Height

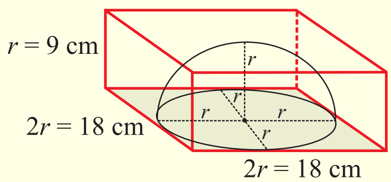
$V = l \times b \times h$

Surface Area $A = 2(lb + bh + lh)$ 12

1 (c) (ii)

You can see from the diagram the dimensions of the rectangular box into which the hemisphere will fit.

$V = l \times b \times h = 18 \times 18 \times 9 = 2916$ cm^3



2005

1 (c) A steel-works buys steel in the form of solid cylindrical rods of radius 10 centimetres and length 30 metres.
The steel rods are melted to produce solid spherical ball-bearings. No steel is wasted in the process.

(i) Find the volume of steel in one cylindrical rod, in terms of π.

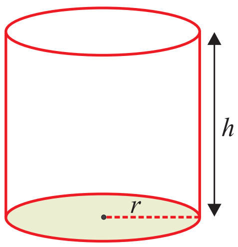
(ii) The radius of a ball-bearing is 2 centimetres.
How many such ball-bearings are made from one steel rod?

(iii) Ball-bearings of a different size are also produced.
One steel rod makes 225 000 of these new ball-bearings.
Find the radius of the new ball-bearings.

SOLUTION

1 (c) (i)

CYLINDER



$V = \pi r^2 h$

Curved SA: $A = 2\pi rh$

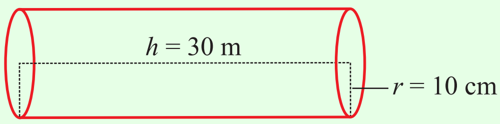
Total SA: $A = 2\pi rh + 2\pi r^2$ **14**

CYLINDRICAL ROD:

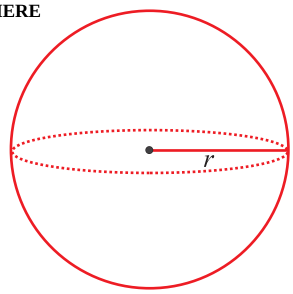
$r = 10$ cm,

$h = 30$ m $= 3000$ cm

$V = \pi r^2 h = \pi(10)^2(3000) = 300,000\pi \text{ cm}^3$



SPHERE



$V = \frac{4}{3}\pi r^3$

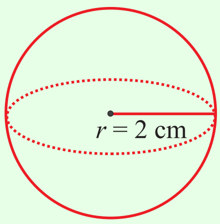
Curved SA: $A = 4\pi r^2$ **15**

1 (c) (ii)

SPHERE:

$r = 2$ cm

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2)^3 = \frac{32}{3}\pi \text{ cm}^3$



RECASTING: These are problems where solids of one type of shape are melted down and recast as solids in another shape. The volume of material in the original shape is the same as the volume in the new shape.

$$\text{Number of ball-bearings} = \frac{\text{Volume of rod}}{\text{Volume of sphere}} = \frac{300000\pi}{\frac{32}{3}\pi} = \frac{300000 \times 3}{32} = 28,125$$

CONT....

1 (c) (iii)
If you divide the volume of the cylindrical rod by 225,000 you get the volume of the new sphere.

Volume of new sphere $= \dfrac{300000\pi}{225000} = \frac{4}{3}\pi \text{ cm}^3$

$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{4}{3}\pi = \frac{4}{3}\pi r^3$

$\Rightarrow 1 = r^3$

$\therefore r = 1 \text{ cm}$

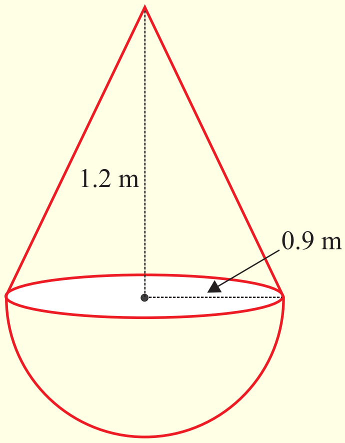
2004

1 (c) A buoy at sea is in the shape of a hemisphere with a cone on top, as in the diagram.
The radius of the base of the cone is 0.9m and its vertical height is 1.2 m.

(i) Find the vertical height of the buoy.

(ii) Find the volume of the buoy, in terms of π.

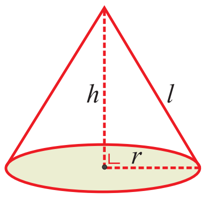
(iii) When the buoy floats, 0.8 m of its height is above water. Find, in terms of π, the volume of that part of the buoy that is above the water.



Solution

1 (c) (i)

Cone



$V = \frac{1}{3}\pi r^2 h$

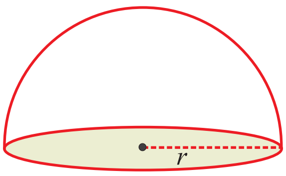
Curved SA: $A = \pi r l$

Total SA: $A = \pi r l + \pi r^2$

.... **17**

You can use Pythagoras on the cone: $l^2 = r^2 + h^2$

Hemisphere



$V = \frac{2}{3}\pi r^3$

Curved SA: $A = 2\pi r^2$

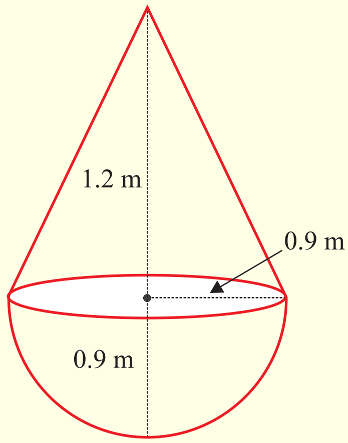
Total SA: $A = 3\pi r^2$

.... **16**

As can be seen from the diagram, the vertical height of the buoy is 1.2 m + 0.9 m = 2.1 m

Cont....

As can be seen from the diagram, the vertical height of the buoy is 1.2 m + 0.9 m = 2.1 m



1 (c) (ii)

Volume of buoy = Volume of cone + Volume of hemisphere

Cone: r = 0.9 m, h = 1.2 m

$$V_1 = \tfrac{1}{3}\pi r^2 h = \tfrac{1}{3}\pi (0.9)^2(1.2) = 0.324\pi \text{ cm}^3$$

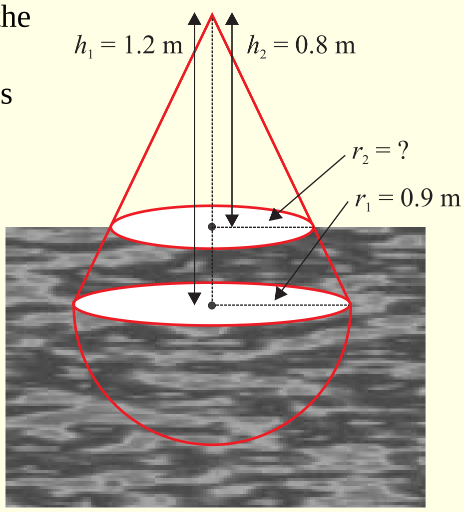
Hemisphere: r = 0.9 m

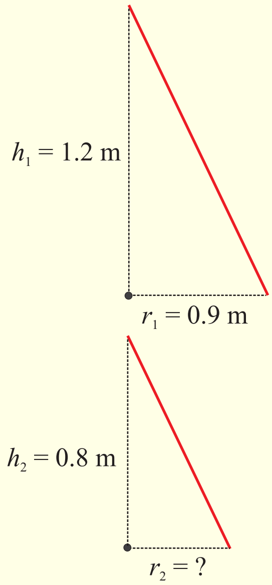
$$V_2 = \tfrac{2}{3}\pi r^3 = \tfrac{2}{3}\pi (0.9)^3 = 0.486\pi \text{ cm}^3$$

Buoy: $V = V_1 + V_2 = 0.324\pi + 0.486\pi = 0.81\pi \text{ cm}^3$

1 (c) (iii)

A cone of height 0.8 m is above the water. What is its radius? Compare the two similar triangles as shown. The ratio of their corresponding sides are equal.





$$\therefore \frac{r_1}{h_1} = \frac{r_2}{h_2} \Rightarrow \frac{r_1}{0.8} = \frac{0.9}{1.2}$$

$$\Rightarrow r_1 = \frac{0.8 \times 0.9}{1.2} = 0.6 \text{ m}$$

Volume of small cone:

$$V = \tfrac{1}{3}\pi r^2 h = \tfrac{1}{3}\pi (0.6)^2 (0.8)$$

$$\therefore V = 0.096\pi \text{ m}^3$$

2003

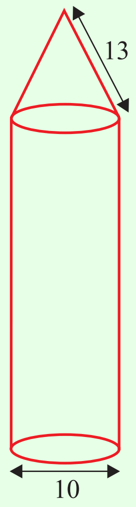
1 (c) A wax crayon is in the shape of a cylinder of diameter 10 mm, surmounted by a cone of slant height 13 mm.

(i) Show that the vertical height of the cone is 12 mm.

(ii) Show that the volume of the cone is 100π mm^3.

(iii) Given that the volume of the cylinder is 15 times the volume of the cone, find the volume of the crayon, in cm^3, correct to two decimal places.

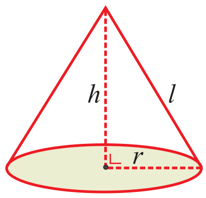
(iv) How many complete crayons like this one can be made from 1 kg of wax, given that each cm^3 of wax weighs 0.75 grammes?



SOLUTION

1 (c) (i)

CONE



$V = \frac{1}{3}\pi r^2 h$

Curved SA: $A = \pi r l$

Total SA: $A = \pi r l + \pi r^2$

.... **17**

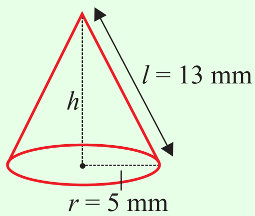
You can use Pythagoras on the cone: $l^2 = r^2 + h^2$

$l^2 = r^2 + h^2 \Rightarrow 13^2 = 5^2 + h^2$

$\Rightarrow h^2 = 169 - 25$

$\Rightarrow h^2 = 144$

$\therefore h = \sqrt{144} = 12$ mm



1 (c) (ii)

$V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi (5)^2 (12)$

$\therefore V = 100\pi$ mm^3

1 (c) (iii)

Volume of cylinder $= 15 \times 100\pi = 1500\pi$ mm^3

Volume of crayon = Volume of cone + Volume of cylinder $= 100\pi + 1500\pi = 1600\pi$ mm^3

$$1 \text{ cm} = 10 \text{ mm}$$
$$1 \text{ cm}^2 = 1 \text{ cm} \times 1 \text{ cm} = 10 \text{ mm} \times 10 \text{ mm} = 100 \text{ mm}^2$$
$$1 \text{ cm}^3 = 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} = 10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm} = 1000 \text{ mm}^3$$

$\therefore$ Volume of crayon $= 1.6\pi$ cm$^3 = 5.03$ cm^3

CONT....

1 (c) (iv)

Weight of one crayon $= 5.03 \times 0.75 \text{ g} = 3.7725 \text{ g}$

How many of these weights are contained in 1 kg (1000 g)?

Number of crayons $= \dfrac{1000 \text{ g}}{3.7725 \text{ g}} = 265$

2002

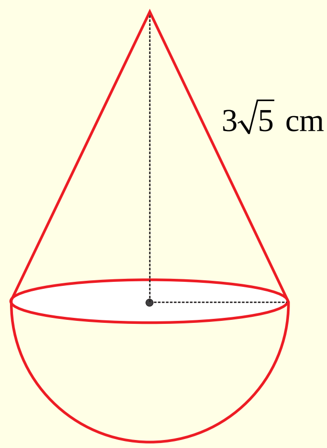
1 (c) A solid is in the shape of a hemisphere surmounted by a cone, as in the diagram.

(i) The volume of the hemisphere is $18\pi \text{ cm}^3$.
Find the radius of the hemisphere.

(ii) The slant height of the cone is $3\sqrt{5}$ cm.
Show that the vertical height of the cone is 6 cm.

(iii) Show that the volume of the cone equals the volume of the hemisphere.

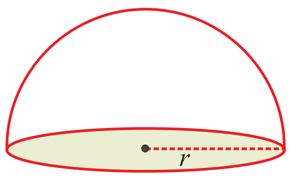
(iv) This solid is melted down and recast in the shape of a solid cylinder.
The height of the cylinder is 9 cm. Calculate its radius.



SOLUTION

1 (c) (i)

HEMISPHERE



$V = \frac{2}{3}\pi r^3$

Curved SA: $A = 2\pi r^2$

Total SA: $A = 3\pi r^2$ **16**

$V = 18\pi \text{ cm}^3$

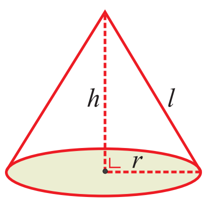
$V = \frac{2}{3}\pi r^3 \Rightarrow 18\pi = \frac{2}{3}\pi r^3$

$\Rightarrow r^3 = \dfrac{3 \times 18}{2} = 27$

$\therefore r = \sqrt[3]{27} = 3 \text{ cm}$

1 (c) (ii)

CONE



$V = \frac{1}{3}\pi r^2 h$

Curved SA: $A = \pi r l$

Total SA: $A = \pi r l + \pi r^2$ **17**

You can use Pythagoras on the cone: $l^2 = r^2 + h^2$

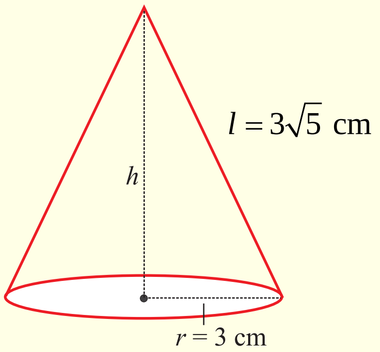
CONT....

$l^2 = r^2 + h^2 \Rightarrow (3\sqrt{5})^2 = 3^2 + h^2$

$\Rightarrow 45 = 9 + h^2$

$\Rightarrow 36 = h^2$

$\therefore h = \sqrt{36} = 6$ cm



1 (c) (iii)

Volume of cone: $r = 3$ cm, $h = 6$ cm

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3)^2(6) = 18\pi \text{ cm}^3$

Volume of hemisphere: $r = 3$ cm

$V = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(3)^3 = 18\pi \text{ cm}^3$

1 (c) (iv)

Recasting: These are problems where solids of one type of shape are melted down and recast as solids in another shape. The volume of material in the original shape is the same as the volume in the new shape.

Volume of solid = Volume of cone + Volume of hemisphere $= 18\pi + 18\pi = 36\pi \text{ cm}^3$

Cylinder: r = ?, h = 9 cm, $V = 36\pi \text{ cm}^3$

$V = \pi r^2 h \Rightarrow 36\pi = \pi r^2 (9)$

$\Rightarrow 36 = 9r^2 \Rightarrow r^2 = 4$

$\therefore r = \sqrt{4} = 2$ cm

2001

1 (c) Sweets, made from a chocolate mixture, are in the shape of solid spherical balls.
The diameter of each sweet is 3 cm.
36 sweets fit exactly in a rectangular box which has internal height 3 cm.

(i) The base of the box is a square. How many sweets are there in each row?

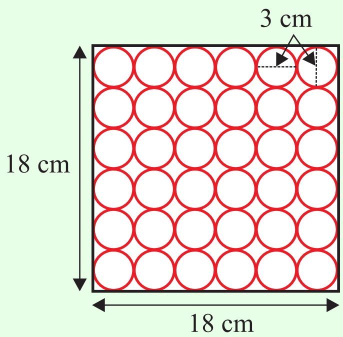
(ii) What is the internal volume of the box?

(iii) The 36 sweets weigh 675 grammes.
What is the weight of 1 cm^3 of the chocolate mixture? Give your answer correct to one decimal place.

SOLUTION

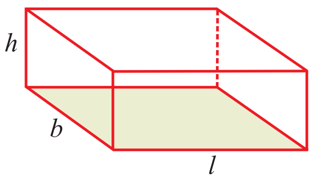
1 (c) (i)

A diagram is drawn looking down on top of the rectangular box with square base.
No. of sweets in each row = 6



1 (c) (ii)

RECTANGULAR SOLID



l: Length
b: Breadth
h: Height

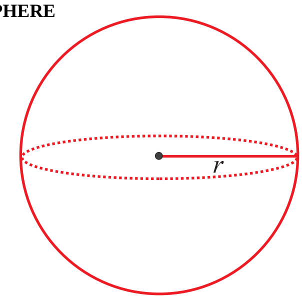
$V = l \times b \times h$

Surface Area $A = 2(lb + bh + lh)$ **12**

$V = l \times b \times h = 18 \times 18 \times 3 = 972 \text{ cm}^2$

1 (c) (iii)

SPHERE



$V = \frac{4}{3}\pi r^3$

Curved SA: $A = 4\pi r^2$ **15**

Calculate the volume of the 36 sweets: $r = \frac{3}{2}$ cm

$V = 36 \times \frac{4}{3}\pi r^3 \Rightarrow V = 36 \times \frac{4}{3}\pi (\frac{3}{2})^3$

$\therefore V = 508.9 \text{ cm}^3$

Weight of 1 cm^3 of mixture $= \dfrac{675}{508.9} \text{ g} = 1.3 \text{ g}$

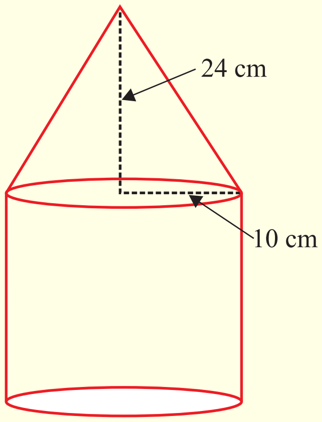
2000

1 (c) A candle is in the shape of a cylinder surmounted by a cone, as in the diagram.

(i) The cone has height 24 cm and the length of the radius of its base is 10 cm.
Find the volume of the cone in terms of π.

(ii) The height of the cylinder is equal to the slant height of the cone.
Find the volume of the cylinder in terms of π.

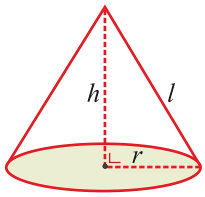
(iii) A solid spherical ball of wax with radius of length r cm was used to make the candle.
Calculate r, correct to one decimal place.



SOLUTION

1 (c) (i)

CONE



$V = \frac{1}{3}\pi r^2 h$

Curved SA: $A = \pi r l$

Total SA: $A = \pi r l + \pi r^2$ **17**

You can use Pythagoras on the cone: $l^2 = r^2 + h^2$

$h = 24$ cm, $r = 10$ cm

$V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi (10)^2 (24)$

$\therefore V = 800\pi \text{ cm}^3$

1 (c) (ii)

Find the slant height of cone.

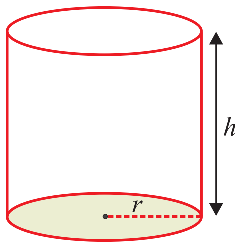
$l^2 = r^2 + h^2 \Rightarrow l^2 = 10^2 + 24^2$

$\Rightarrow l^2 = 100 + 576 = 676$

$\therefore l = \sqrt{676} = 26$ cm

Therefore, the height of the cylinder is 26 cm.

CYLINDER



$V = \pi r^2 h$

Curved SA: $A = 2\pi r h$

Total SA: $A = 2\pi r h + 2\pi r^2$ **14**

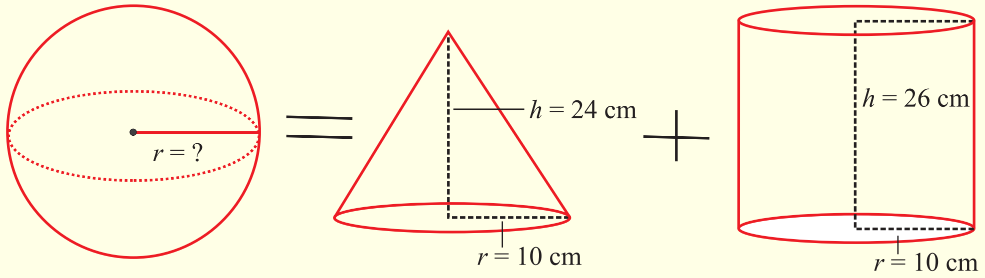
Cylinder: $h = 26$ cm, $r = 10$ cm

$V = \pi r^2 h \Rightarrow V = \pi (10)^2 (26)$

$\therefore V = 2600\pi \text{ cm}^3$

CONT....

1 (c) (iii)



Volume of sphere = Volume of cone + Volume of cylinder $= 800\pi + 2600\pi = 3400\pi \text{ cm}^3$

Volume of sphere: $V = \frac{4}{3}\pi r^3$

$\therefore 3400\pi = \frac{4}{3}\pi r^3 \Rightarrow \dfrac{3 \times 3400}{4} = r^3$

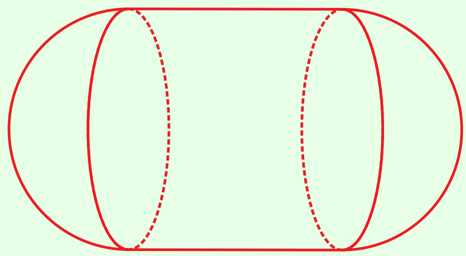
$\Rightarrow r^3 = 2550$

$\therefore r = \sqrt[3]{2550} = 13.7 \text{ cm}$

1999

1 (c) (i) Write down, in terms of π and r, the volume of a hemisphere with radius of length r.

(ii) A fuel storage tank is in the shape of a cylinder with a hemisphere at each end, as shown.



The capacity (internal volume) of the tank is $81\pi \text{ m}^3$.

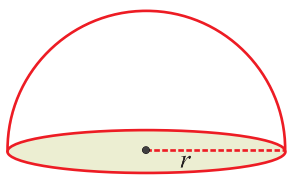
The ratio of the capacity of the cylindrical section to the sum of the capacities of the hemispherical ends 5:4.

Calculate the internal radius length of the tank.

SOLUTION

1 (c) (i)

HEMISPHERE



$V = \frac{2}{3}\pi r^3$

Curved SA: $A = 2\pi r^2$

Total SA: $A = 3\pi r^2$

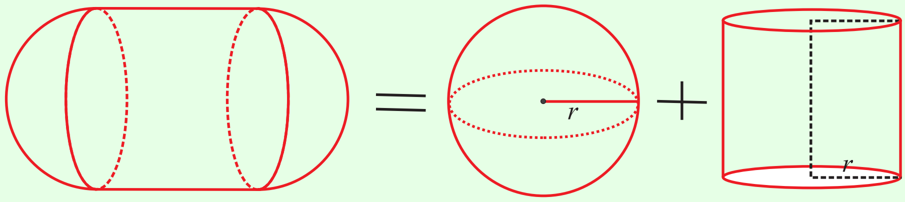
.... **16**

$V = \frac{2}{3}\pi r^3$

1 (c) (ii)

The tank is made up of 2 hemispheres (i.e. one sphere) and a cylinder. The radius of the cylinder and the sphere is the same.

CONT....



Volume of tank $= 81\pi \text{ m}^3$

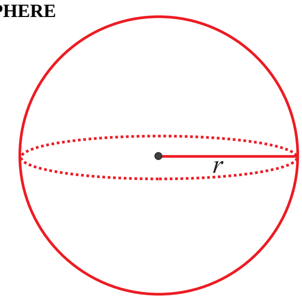
Ratio of the volume of the cylinder to the sphere is 5:4.

5 + 4 = 9.

Therefore, the volume of the sphere is $\frac{4}{9}$ of the overall volume.

Volume of the sphere $= \frac{4}{9} \times 81\pi = 36\pi \text{ m}^3$

SPHERE



$V = \frac{4}{3}\pi r^3$

Curved SA: $A = 4\pi r^2$ **15**

Sphere:

$V = \frac{4}{3}\pi r^3 \Rightarrow 36\pi = \frac{4}{3}\pi r^3$

$\Rightarrow \dfrac{36 \times 3}{4} = r^3$

$\Rightarrow r^3 = 27$

$\therefore r = \sqrt[3]{27} = 3 \text{ cm}$

1998

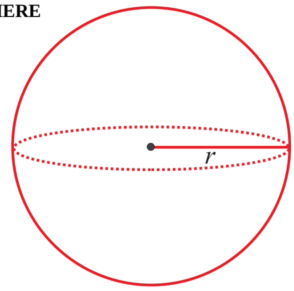
1 (c) Find the volume of a solid sphere with a diameter of length 3 cm. Give your answer in terms of π.

A cylindrical vessel with internal diameter of length 15 cm contains water. The surface of the water is 11 cm from the top of the vessel.

How many solid spheres, each with diameter of length 3 cm, must be placed in the vessel in order to bring the surface of the water to 1 cm from the top of the vessel?
Assume that all the spheres are submerged in the water.

SOLUTION

SPHERE



$V = \frac{4}{3}\pi r^3$

Curved SA: $A = 4\pi r^2$

.... **15**

Sphere:

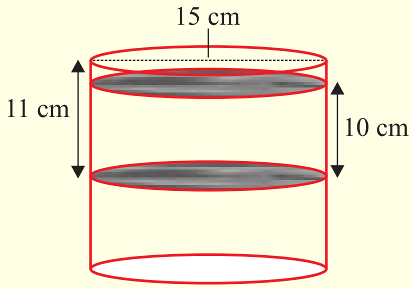
$r = \frac{3}{2}$ cm

$V = \frac{4}{3}\pi r^3 \Rightarrow V = \frac{4}{3}\pi(\frac{3}{2})^3$

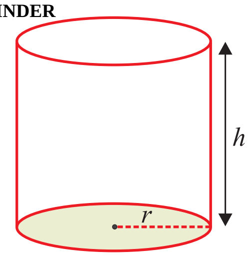
$\Rightarrow V = \frac{4}{3}\pi(\frac{27}{8})$

$\therefore V = \frac{9}{2}\pi \text{ cm}^3$

When the spheres are submerged in the water they raise the height of the water. The spheres displace of volume of water in the cylinder of height 10 cm. Find the volume of a cylinder of height 10 cm.



CYLINDER



$V = \pi r^2 h$

Curved SA: $A = 2\pi rh$

Total SA: $A = 2\pi rh + 2\pi r^2$

.... **14**

CYLINDER:

$r = \frac{15}{2}$ cm, $h = 10$ cm

$V = \pi r^2 h \Rightarrow V = \pi(\frac{15}{2})^2(10)$

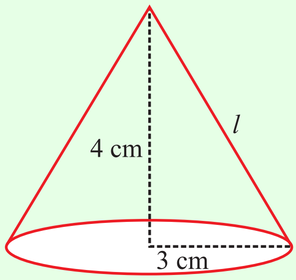
$\therefore V = \frac{1125}{2}\pi \text{ cm}^3$

Divide the volume of a sphere into this volume to find the number of spheres submerged.

Number of spheres $= \dfrac{\frac{1125}{2}\pi}{\frac{9}{2}\pi} = 125$

1997

1 (a) Find the slant height, l, of a cone which has perpendicular height of 4 cm and base with radius of length 3 cm.
Write down the curved surface area of the cone in terms of π.



1 (c) Find the volume of a solid sphere which has radius of length 2.1 cm. Give your answer correct to the nearest cm^3. Take $\frac{22}{7}$ as an approximation of π.

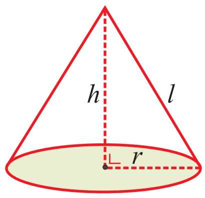
This sphere and a solid cube with edge of length 3 cm are completely submerged in water in a cylinder. The cylinder has radius of length r cm.

Both the sphere and the cube are then removed from the cylinder. The water level drops by 4 cm. Find r, correct to one place of decimals. [Take $\pi = \frac{22}{7}$.]

Solution

1 (a)

Cone



$V = \frac{1}{3}\pi r^2 h$

Curved SA: $A = \pi r l$

Total SA: $A = \pi r l + \pi r^2$

.... **17**

You can use Pythagoras on the cone: $l^2 = r^2 + h^2$

$h = 4$ cm, $r = 3$ cm

$l^2 = r^2 + h^2 \Rightarrow l^2 = 3^2 + 4^2$

$\Rightarrow l^2 = 9 + 16 = 25$

$\therefore l = \sqrt{25} = 5$ cm

Curved Surface Area:

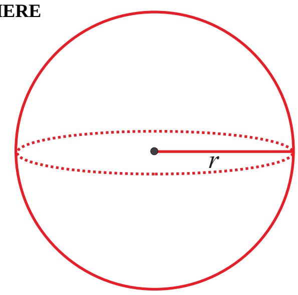
$A = \pi r l \Rightarrow A = \pi(3)(5)$

$\therefore A = 15\pi \text{ cm}^2$

Cont....

1 (c)

SPHERE



$V = \frac{4}{3}\pi r^3$

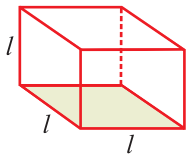
Curved SA: $A = 4\pi r^2$ **15**

$r = 2.1$ cm, $\pi = \frac{22}{7}$

$V = \frac{4}{3}\pi r^3 \Rightarrow V = \frac{4}{3}(\frac{22}{7})(2.1)^3$

$\therefore V = 39 \text{ cm}^3$

CUBE

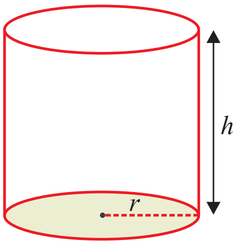


l: Length

$V = l^3$

Surface Area $A = 6l^2$ **13**

CYLINDER



$V = \pi r^2 h$

Curved SA: $A = 2\pi rh$

Total SA: $A = 2\pi rh + 2\pi r^2$ **14**

Firstly, find the total volume of the sphere and cube.

CUBE: $l = 3$ cm

$V = l^3 \Rightarrow V = (3)^3 = 27 \text{ cm}^3$

Total volume of sphere and cube = 39 + 27 = 66 cm^3

When the sphere and cube are removed from a cylinder of water, the height h falls by 4 cm. The volume of this cylinder of water equals the volume of the sphere and cube.

r = ?, h = 4 cm, V = 66 cm^3

$V = \pi r^2 h \Rightarrow 66 = (\frac{22}{7})r^2(4)$

$\Rightarrow r^2 = \frac{66 \times 7}{22 \times 4} = \frac{21}{4}$

$\therefore r = \sqrt{\frac{21}{4}} = 2.3$ cm

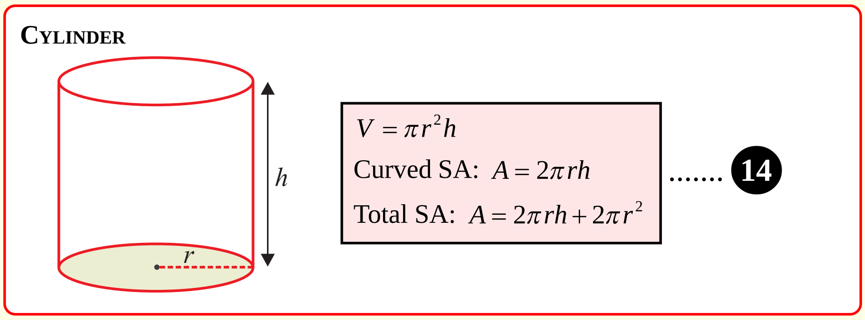
1996

1 (c) A solid cylinder, made of lead, has a radius of length 15 cm and height of 135 cm. Find its volume in terms of π.

The solid cylinder is melted down and recast to make four identical right circular solid cones. The height of each cone is equal to twice the length of its base radius.

Calculate the base radius length of the cones.

SOLUTION

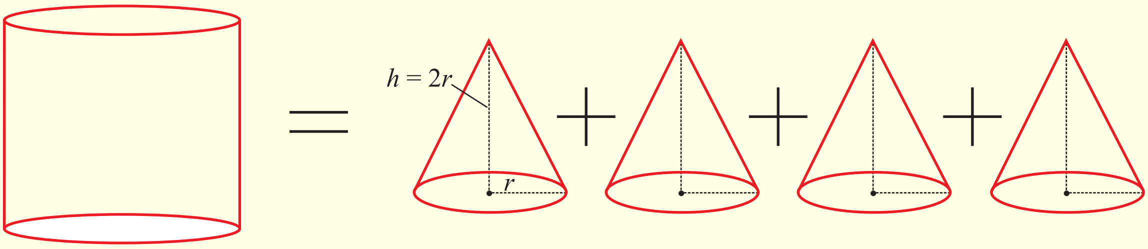


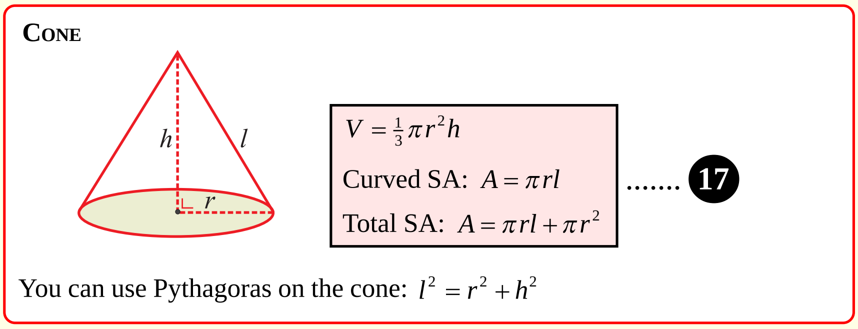
CYLINDER: r = 15 cm, h = 135 cm

$V = \pi r^2 h \Rightarrow V = \pi (15)^2 (135)$

$\therefore V = 30375\pi \text{ cm}^3$

RECASTING: These are problems where solids of one type of shape are melted down and recast as solids in another shape. The volume of material in the original shape is the same as the volume in the new shape.





CONE: $r = r$, $h = 2r$

$V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi r^2 (2r)$

$\therefore V = \frac{2}{3}\pi r^3$

Volume of cylinder = 4 times the volume of the cone

$\therefore 30375\pi = 4 \times \frac{2}{3}\pi r^3$

$\Rightarrow \frac{30375 \times 3}{8} = r^3$

$\therefore r = \sqrt[3]{\frac{30375 \times 3}{8}} = 22.5 \text{ cm}$