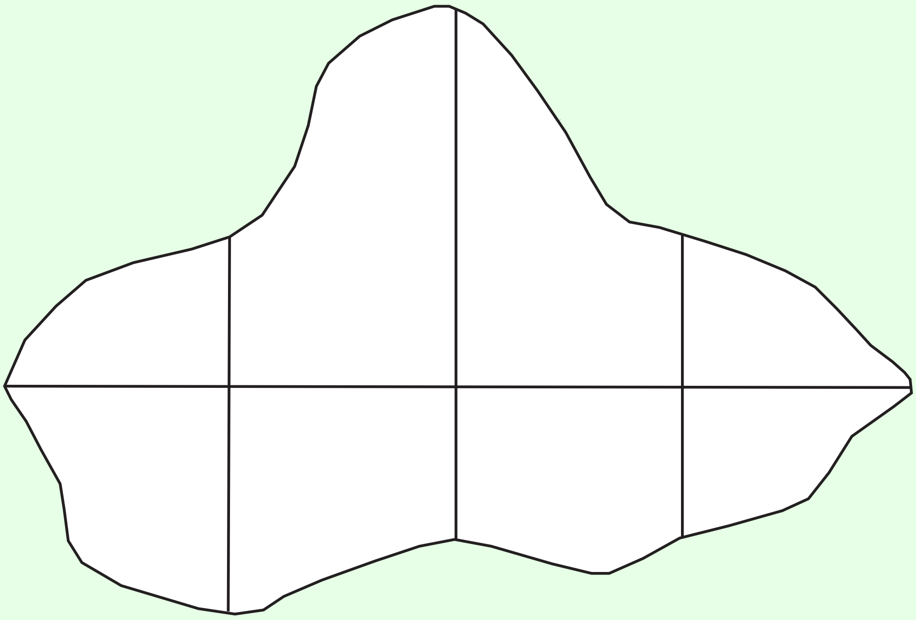
AREA & VOLUME (Q 1, PAPER 2)

LESSON NO. 2: SIMPSON'S RULE

2007

1 (b) In order to estimate the area of the irregular shape below, a horizontal line is drawn across the widest part of the shape and three offsets (perpendicular lines) are drawn at equal intervals along this line.

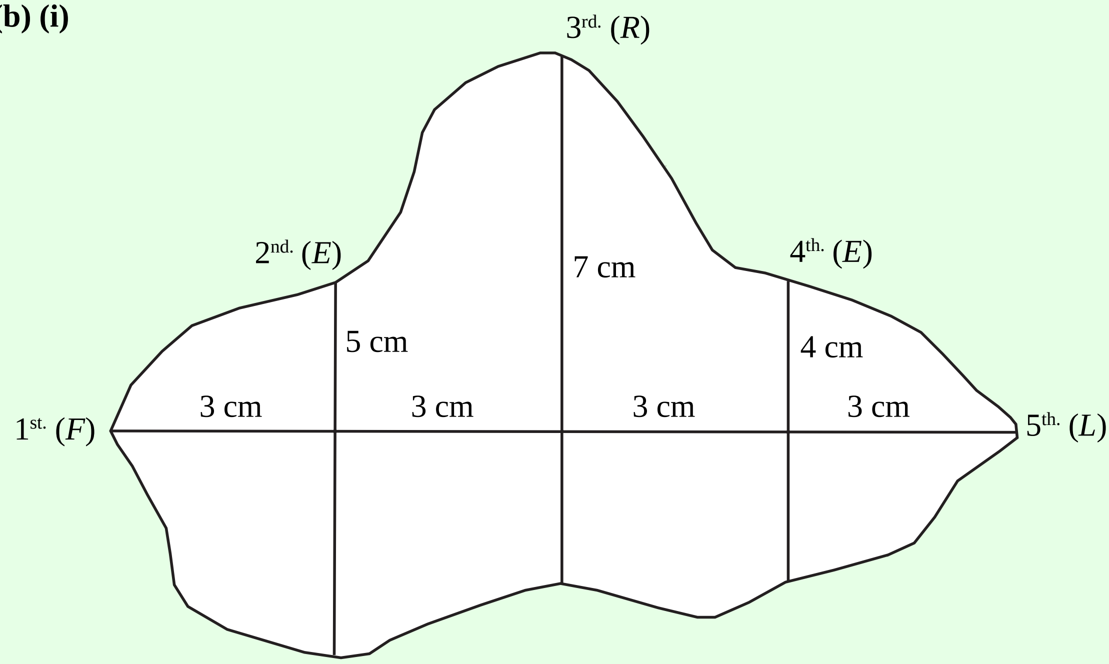


(i) Measure the horizontal line and the offsets, in centimetres.
Make a rough sketch of the shape in your answerbook and record the measurements on it.

(ii) Use Simpson's Rule with these measurements to estimate the area of the shape.

SOLUTION

1 (b) (i)



CONT....

1 (b) (ii)

$A \approx \frac{h}{3}$ [(First + Last) + 4(Evens) + 2(Remaining Odds)]...... **11**

$h = 3$ cm
First = Last = 0 cm

$A = \frac{3}{3}[(0+0)+4(5+4)+2(7)]$

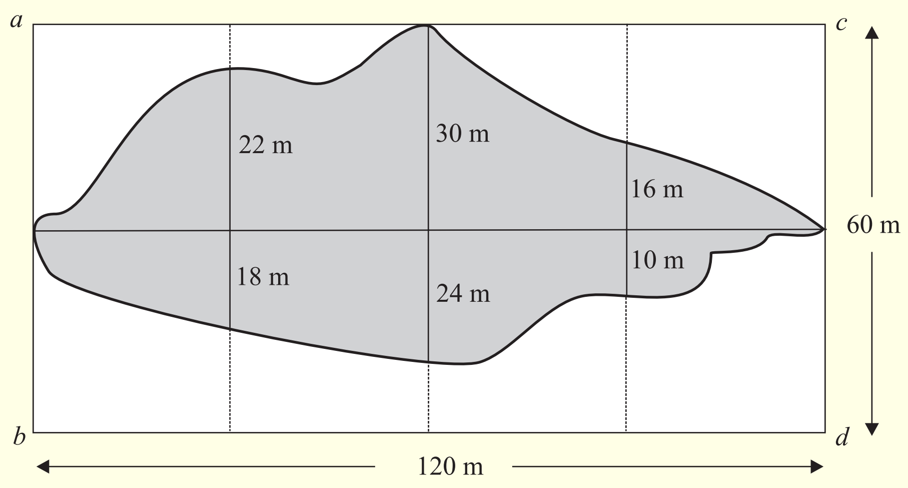
$\Rightarrow A = 1[4(9)+2(7)]$

$\Rightarrow A = [36+14]$

$\therefore A = 50 \text{ cm}^2$

2006

1 (b) Archaeologists excavating a rectangular plot *abcd* measuring 120 m by 60 m divided the plot into eight square sections as shown on the diagram. At the end of the first phase of the work the shaded area had been excavated. To estimate the area excavated, perpendicular measurements were made to the edge of the excavated area, as shown.

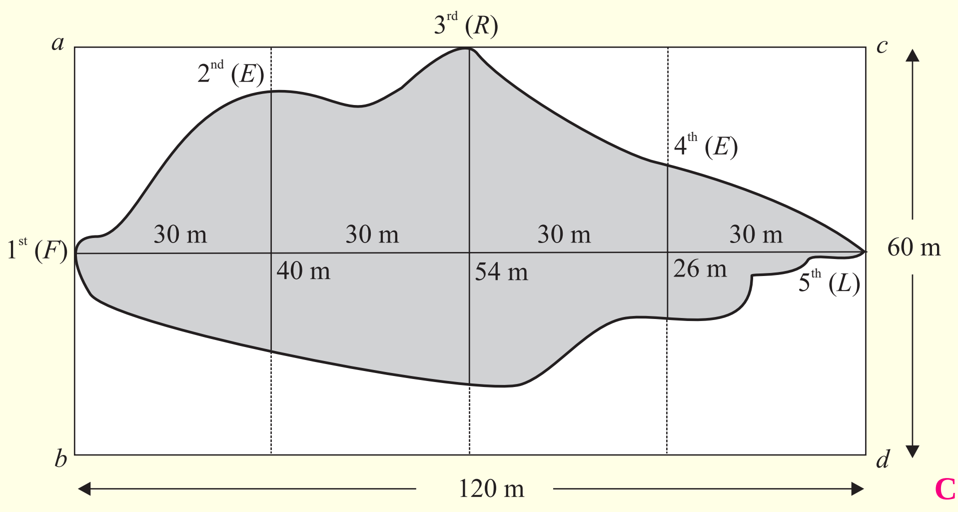


(i) Use Simpson's Rule to estimate the area excavated.

(ii) Express the excavated area as a percentage of the total area, correct to the nearest whole number.

SOLUTION

1 (b) (i)



CONT....

$A \approx \frac{h}{3}$ [(First + Last) + 4(Evens) + 2(Remaining Odds)]...... 11

$h = 120 \div 4 = 30$ m

$A = \frac{30}{3}[(0+0)+4(40+26)+2(54)]$

$\Rightarrow A = 10[0+4(66)+108]$

$\Rightarrow A = 10[264+108]$

$\therefore A = 10[372] = 3720 \text{ cm}^2$

1 (b) (ii)

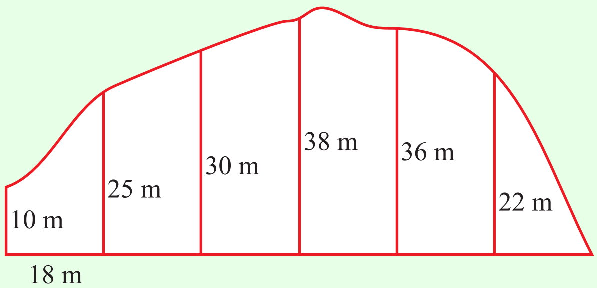
Total Area $= 120 \times 60 = 7200 \text{ m}^2$

Excavated Area = 3720 m^2

$\therefore \frac{\text{Excavated Area}}{\text{Total Area}} \times 100\% = \frac{3720}{7200} \times 100\% = 52\%$

2005

1 (b) The sketch shows a lake bounded on one side by a straight dam.



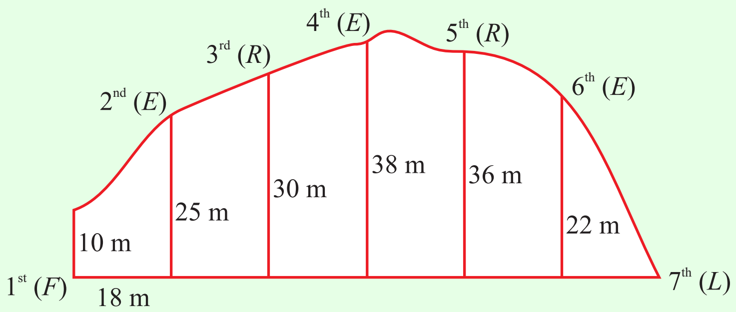
At equal intervals of 18 m along the dam, perpendicular measurements are made to the opposite bank, as shown on the sketch.

(i) Use Simpson's Rule to estimate the area of the lake.

(ii) If the lake contains 15 000 m^3 of water, calculate the average depth of water in the lake, correct to the nearest metre.

SOLUTION

1 (b) (i)



$h = 18$ $A \approx \frac{h}{3}$ [(First + Last) + 4(Evens) + 2(Remaining Odds)]...... 11

$A = \frac{18}{3}[(10+0)+4(25+38+22)+2(30+36)]$

$\Rightarrow A = 6[(10)+4(85)+2(66)]$

$\Rightarrow A = 6[10+340+132]$

$\therefore A = 6[482] = 2892 \text{ m}^2$

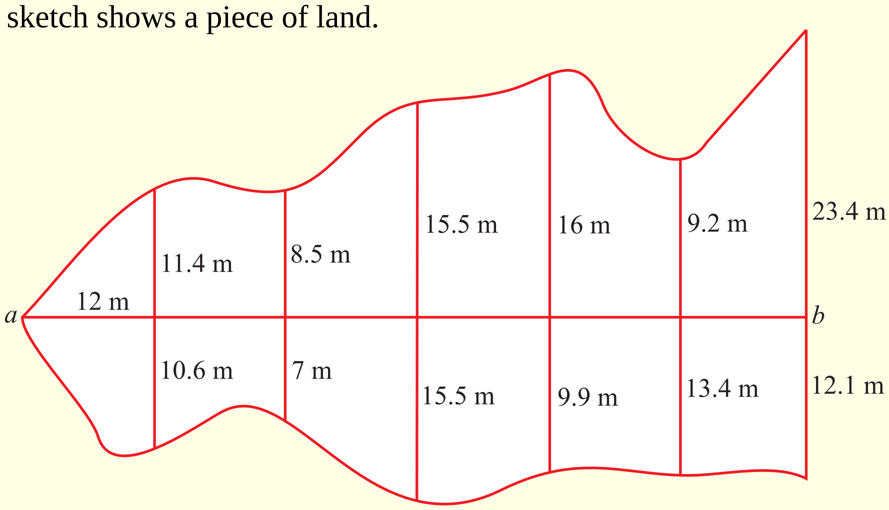
CONT....

1 (b) (ii)

$$\text{Volume} = \text{Area} \times \text{Depth} \Rightarrow \text{Depth} = \frac{\text{Volume}}{\text{Area}} = \frac{15000}{2892} = 5 \text{ m}$$

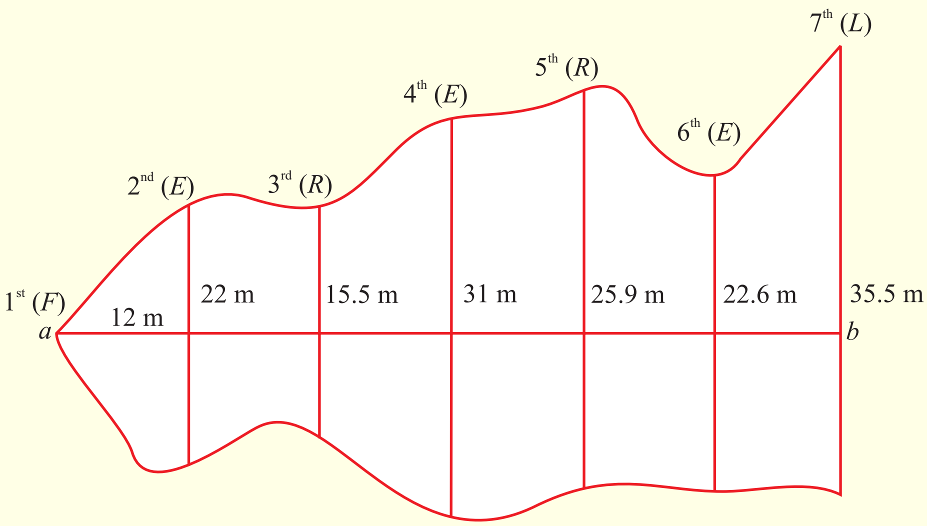
2004

1 (b) The sketch shows a piece of land.



At equal intervals of 12 m along [*ab*], perpendicular measurements are made to the boundary, as shown on the sketch.
Use Simpson's Rule to estimate the area of the piece of land.

Solution



$A \approx \frac{h}{3}$ [(First + Last) + 4(Evens) + 2(Remaining Odds)] **11**

$A \approx \frac{12}{3}[(0+35.5)+4(22+31+22.6)+2(15.5+25.9)]$

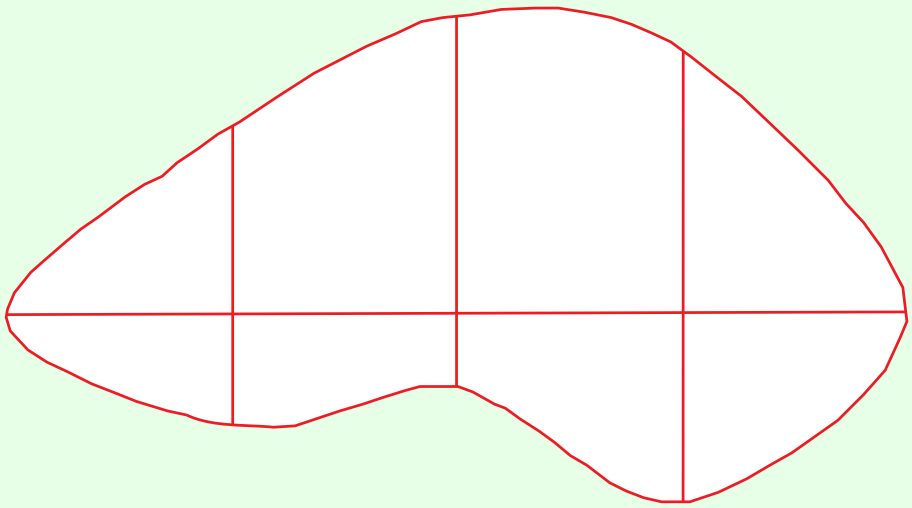
$\Rightarrow A \approx 4[(35.5)+4(75.6)+2(41.4)]$

$\Rightarrow A \approx 4[35.5+302.4+82.8]$

$\Rightarrow A \approx 4[420.7] = 1682.8 \text{ cm}^2$

2003

1 (b) In order to estimate the area of the irregular shape below, a horizontal line is drawn across the widest part of the shape and three offsets (perpendicular lines) are drawn at equal intervals along this line.

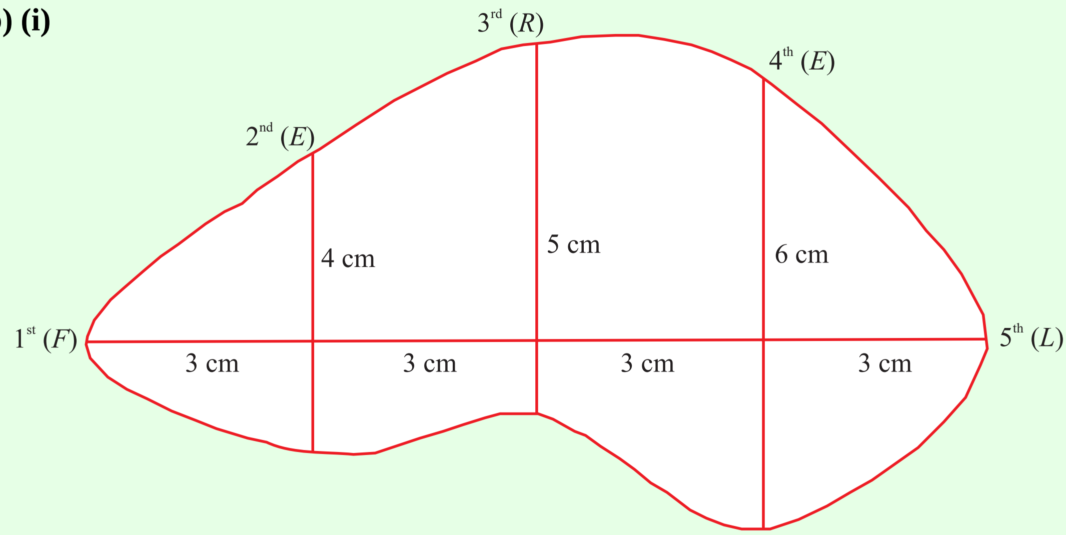


(i) Measure the horizontal line and the offsets as accurately as you can. Make a rough sketch of the shape in your answerbook and record the measurements on it.

(ii) Use Simpson's Rule with these measurements to estimate the area of the shape.

SOLUTION

1 (b) (i)



1 (b) (ii)

$A \approx \frac{h}{3}$ [(First + Last) + 4(Evens) + 2(Remaining Odds)] **11**

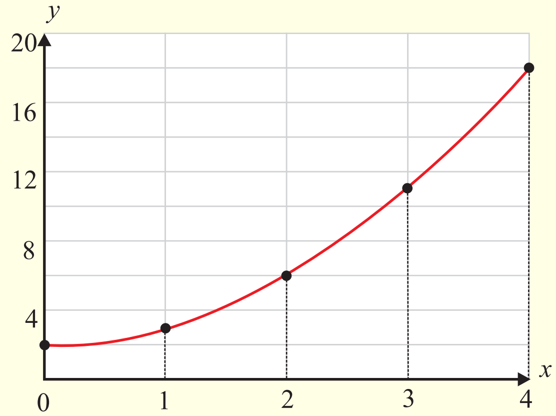
$A \approx \frac{3}{3}[(0+0)+4(4+6)+2(5)]$

$\Rightarrow A \approx 1[0+4(10)+10]$

$\therefore A \approx [40+10] = 50 \text{ cm}^2$

2002

1 (b) The diagram shows the curve $y = x^2 + 1$ in the domain $0 \le x \le 4$.



(i) Copy the following table. Then, complete it using the equation of the curve:

x	0	1	2	3	4
y					

(ii) Hence, use Simpson's Rule to estimate the area between the curve and the x-axis.

SOLUTION

1 (b) (i)

$x = 0: y = (0)^2 + 1 = 0 + 1 = 1$

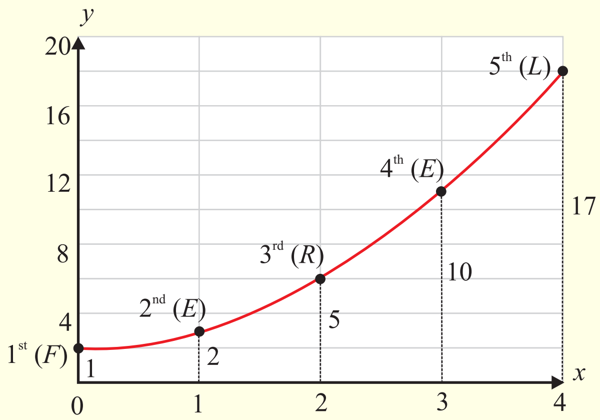
$x = 1: y = (1)^2 + 1 = 1 + 1 = 2$

$x = 2: y = (2)^2 + 1 = 4 + 1 = 5$

$x = 3: y = (3)^2 + 1 = 9 + 1 = 10$

$x = 4: y = (4)^2 + 1 = 16 + 1 = 17$

x	0	1	2	3	4
y	1	2	5	10	17



1 (b) (ii)

$A \approx \frac{h}{3}$ [(First + Last) + 4(Evens) + 2(Remaining Odds)] **11**

$h = 1$ unit

$A \approx \frac{1}{3}[(1+17) + 4(2+10) + 2(5)]$

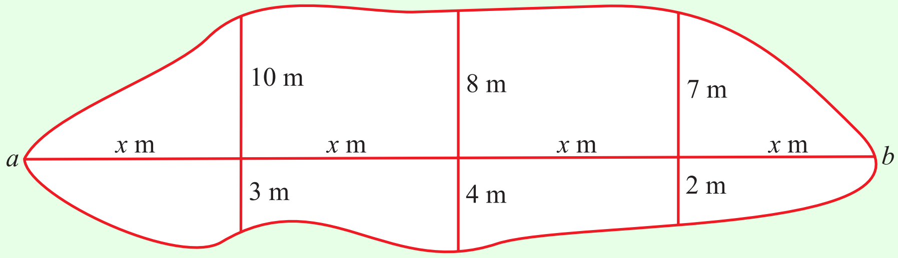
$\Rightarrow A \approx \frac{1}{3}[(18) + 4(12) + 2(5)]$

$\Rightarrow A \approx \frac{1}{3}[18 + 48 + 10]$

$\therefore A \approx \frac{1}{3}[76] = \frac{76}{3}$ units^2

2001

1 (b) The sketch shows a flood caused by a leaking underground pipe that runs from *a* to *b*.

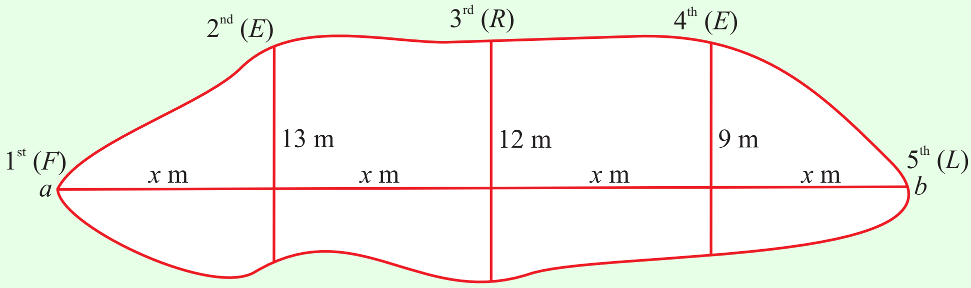


At equal intervals of *x* m along [*ab*] perpendicular measurements are made to the edges of the flood. The measurements to the top edge are 10 m, 8 m and 7 m. The measurements to the bottom edge are 3 m, 4 m and 2 m. At *a* and *b* the measurements are 0 m.

Using Simpson's Rule the area of the flood is estimated to be 672 m^2.

Find *x* and hence, write down the length of the pipe.

SOLUTION



$A \approx \frac{h}{3}$ [(First + Last) + 4(Evens) + 2(Remaining Odds)] **11**

$h = x, \ A = 672 \text{ m}^2$

$672 \approx \frac{x}{3}[(0+0)+4(13+9)+2(12)]$

$\Rightarrow 672 \approx \frac{x}{3}[0+4(22)+2(12)]$

$\Rightarrow 672 \approx \frac{x}{3}[88+24]$

$\Rightarrow 672 \approx \frac{x}{3}[112]$

$\Rightarrow \dfrac{672 \times 3}{112} \approx x$

$\therefore x \approx 18 \text{ m}$

Length of pipe $= 4 \times 18 \text{ m} = 72 \text{ m}$

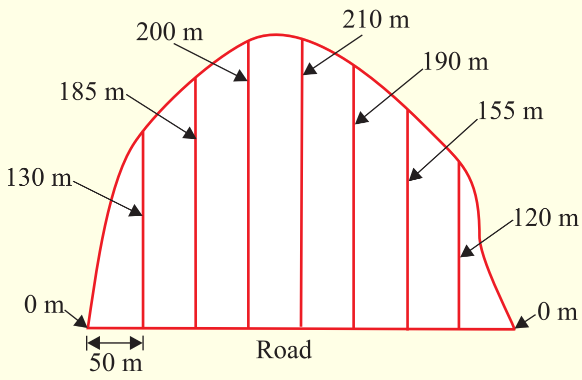
2000

1 (b) The sketch shows a piece of land covered by forest which lies on one side of a straight road.

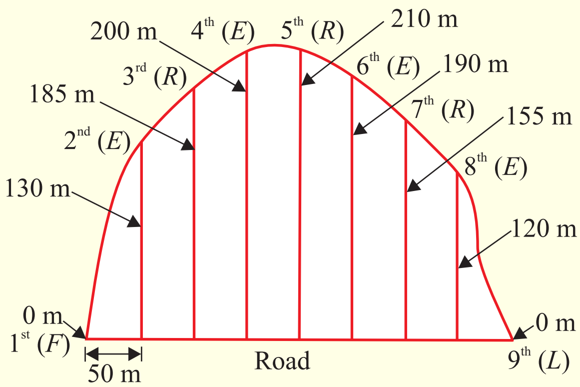
At equal intervals of 50 m along the road, perpendicular measurements of 130 m, 185 m, 200 m, 210 m, 190 m, 155 m and 120 m are made to the forest boundary.

Use Simpson's Rule to estimate the area of land covered by the forest. [See Tables, page 42.]

Give your answer in hectares. [Note: 1 hectare = 10 000 m^2.]



Solution



$A \approx \frac{h}{3}$ [(First + Last) + 4(Evens) + 2(Remaining Odds)] **11**

$h = 50$ m

$A \approx \frac{50}{3}[(0+0)+4(130+200+190+120)+2(185+210+155)]$

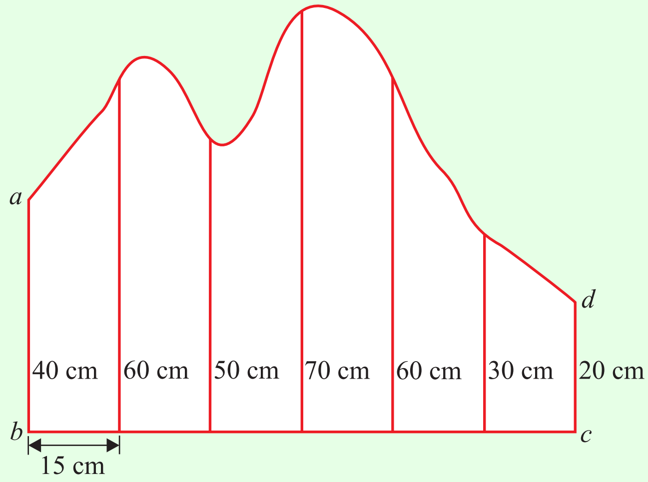
$\Rightarrow A \approx \frac{50}{3}[0+4(640)+2(550)]$

$\Rightarrow A \approx \frac{50}{3}[2560+1100]$

$\therefore A \approx \frac{50}{3}[3660] = 61{,}000 \text{ m}^2 = 6.1$ hectares

1999

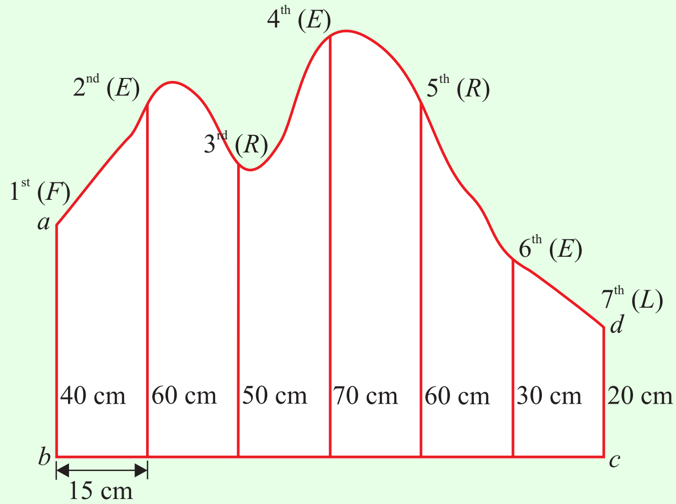
1 (b) A sketch of a piece of land *abcd* is shown.



At equal intervals of 15 m along [*bc*], perpendicular measurements of 40 m, 60 m, 50 m, 70 m, 60 m, 30 m and 20 m are made to the top boundary.

Use Simpson's Rule to estimate the area of the piece of land. [See Tables, page 42].

SOLUTION



$A \approx \frac{h}{3}$ [(First + Last) + 4(Evens) + 2(Remaining Odds)] **11**

$h = 15$ cm

$A \approx \frac{15}{3}[(40+20)+4(60+70+30)+2(50+60)]$

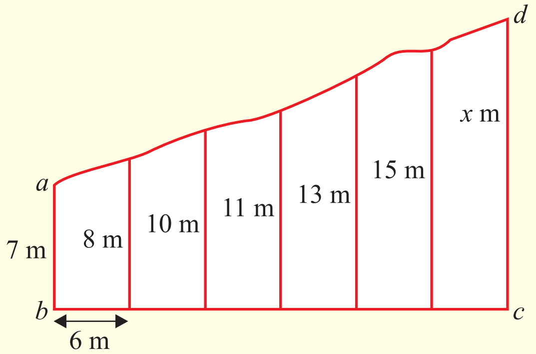
$\Rightarrow A \approx 5[(60)+4(160)+2(110)]$

$\Rightarrow A \approx 5[60+640+220]$

$\therefore A \approx 5[920] = 4600 \text{ cm}^2$

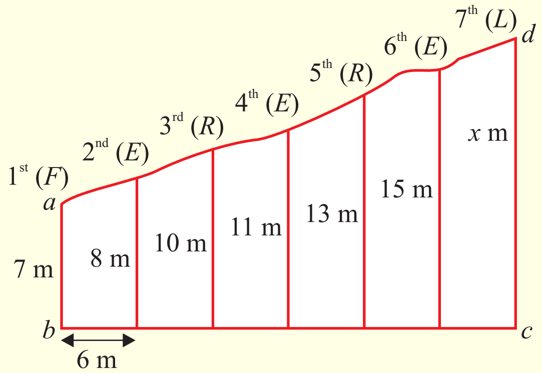
1998

1 (b) The sketch shows a field *abcd* which has one uneven edge. At equal intervals of 6 m along [*bc*] perpendicular measurements of 7 m, 8 m, 10 m, 11 m, 13 m, 15 m and *x* m are made to the top of the field.



Using Simpson's Rule the area of the field is calculated to be 410 m^2.
Calculate the value of x. [See Tables, page 42.]

SOLUTION



$A \approx \frac{h}{3}$ [(First + Last) + 4(Evens) + 2(Remaining Odds)] **11**

$h = 6$ m, $A = 410$ m^2

$410 = \frac{6}{3}[(7+x)+4(8+11+15)+2(10+13)]$

$\Rightarrow 410 = 2[(7+x)+4(34)+2(23)]$

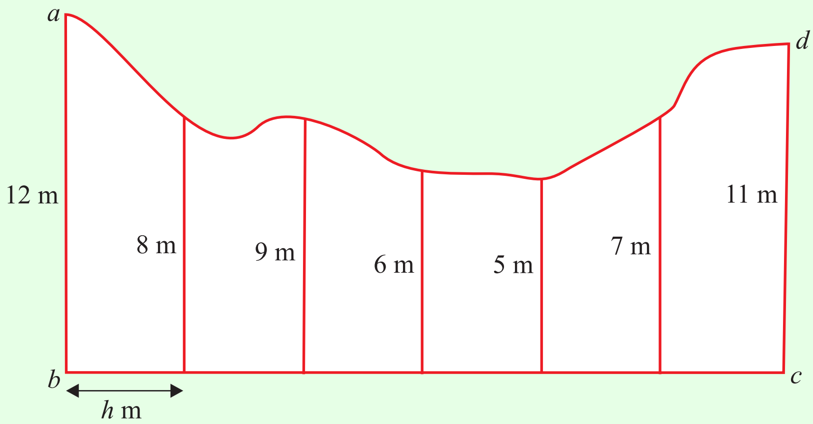
$\Rightarrow 410 = 2[7+x+136+46]$

$\Rightarrow 205 = [x+189]$

$\therefore x = 205-189 = 16$ m

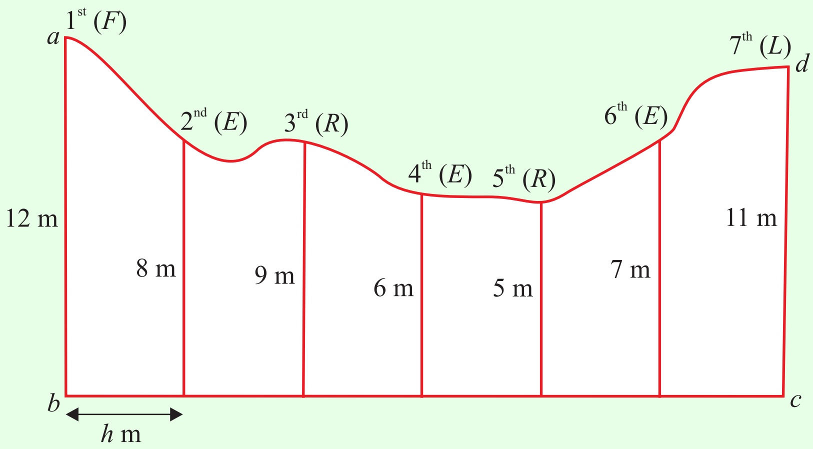
1997

1 (b) The diagram shows a sketch of a piece of paper abcd with one uneven edge. At equal intervals of h cm along [bc], perpendicular measurements of 12 cm, 8 cm, 9 cm, 6 cm, 5 cm, 7 cm and 11 cm are made to the top edge.



Use Simpson's Rule the area of the piece of paper is estimated to be 180 cm^2. Calculate the value of h. [See Tables, page 42.]

SOLUTION



$A \approx \frac{h}{3}$[(First + Last) + 4(Evens) + 2(Remaining Odds)] **11**

$A = 180 \text{ cm}^2$

$180 = \frac{h}{3}[(12+11)+4(8+6+7)+2(9+5)]$

$\Rightarrow 180 = \frac{h}{3}[(23)+4(21)+2(14)]$

$\Rightarrow 180 = \frac{h}{3}[23+84+28]$

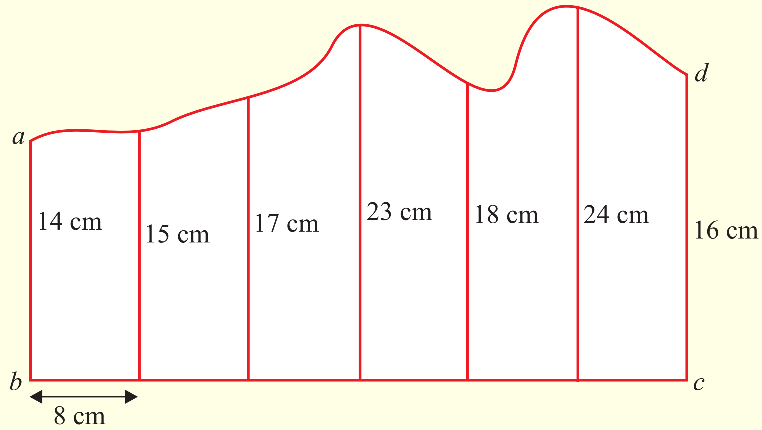
$\Rightarrow 180 = \frac{h}{3}[135]$

$\Rightarrow 180 = h[45]$

$\therefore h = \frac{180}{45} = 4 \text{ cm}$

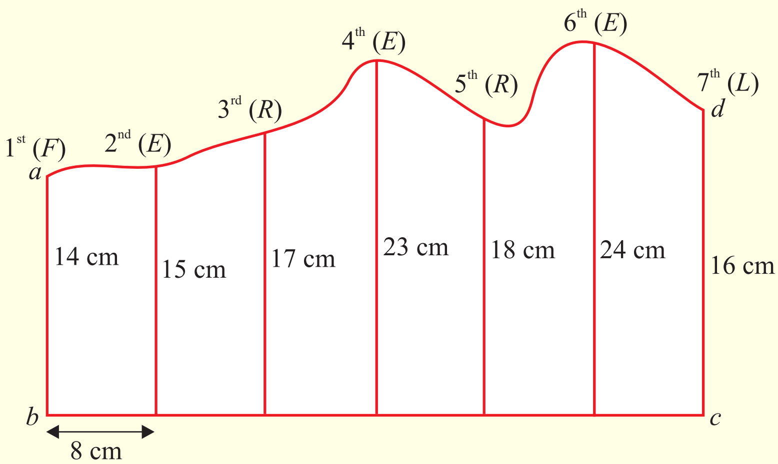
1996

1 (b) A sketch to estimate the area of a building site abcd is shown. At intervals of 8 m along [*bc*], perpendicular measurements of 14 m, 15 m, 17 m, 23 m, 18 m, 24 m and 16 m are made to the top boundary.



Use Simpson's Rule to estimate the area of the building site. [See Tables, page 42].

SOLUTION



$A \approx \frac{h}{3}$ [(First + Last) + 4(Evens) + 2(Remaining Odds)] **11**

$h = 8$ cm

$A \approx \frac{8}{3}[(14+16)+4(15+23+24)+2(17+18)]$

$\Rightarrow A \approx \frac{8}{3}[(30)+4(62)+2(35)]$

$\Rightarrow A \approx \frac{8}{3}[30+248+70]$

$\therefore A \approx \frac{8}{3}[348] = 928 \text{ cm}^3$