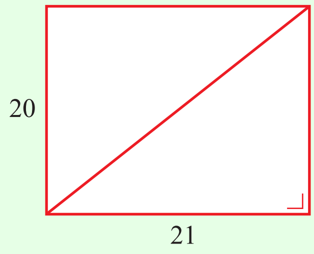
AREA & VOLUME (Q 1, PAPER 2)

2005

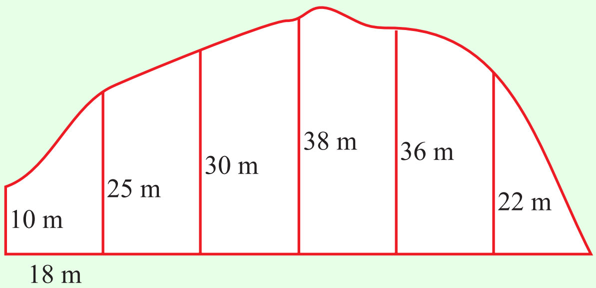
1 (a) A rectangle has length 21 cm and width 20 cm.

(i) Find the area of the rectangle.

(ii) Find the length of the diagonal.



(b) The sketch shows a lake bounded on one side by a straight dam.



At equal intervals of 18 m along the dam, perpendicular measurements are made to the opposite bank, as shown on the sketch.

(i) Use Simpson's Rule to estimate the area of the lake.

(ii) If the lake contains 15 000 m^3 of water, calculate the average depth of water in the lake, correct to the nearest metre.

(c) A steel-works buys steel in the form of solid cylindrical rods of radius 10 centimetres and length 30 metres.
The steel rods are melted to produce solid spherical ball-bearings. No steel is wasted in the process.

(i) Find the volume of steel in one cylindrical rod, in terms of π.

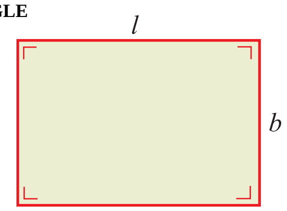
(ii) The radius of a ball-bearing is 2 centimetres.
How many such ball-bearings are made from one steel rod?

(iii) Ball-bearings of a different size are also produced.
One steel rod makes 225 000 of these new ball-bearings.
Find the radius of the new ball-bearings.

Solution

1 (a) (i)

1. **Rectangle**



l: Length
b: Breadth

$A = l \times b$
$P = 2l + 2b = 2(l + b)$ **1**

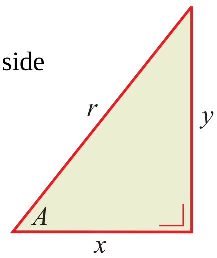
$l = 21$ cm, $b = 20$ cm

$A = l \times b = 21 \times 20 = 420 \text{ cm}^2$

3. **Right-Angled Triangles**

Pythagoras

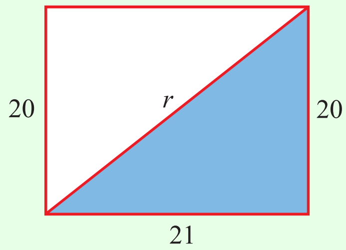
One of the angles in a right-angled triangle is 90°. The side opposite this angle is called the **hypotenuse**.
Pythagoras' theorem applies to right-angled triangles.



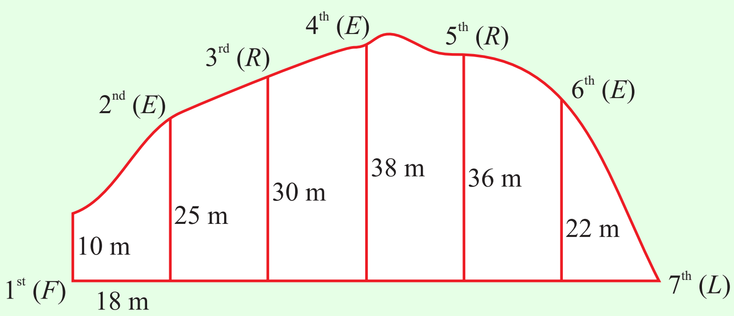
$x^2 + y^2 = r^2$ **3**

1 (a) (ii)

$$r^2 = 20^2 + 21^2$$
$$\Rightarrow r^2 = 400 + 441 = 841$$
$$\therefore r = \sqrt{841} = 29 \text{ cm}$$



1 (b) (i)



$A \approx \frac{h}{3}$[(First + Last) + 4(Evens) + 2(Remaining Odds)] **11**

$h = 18$

$A = \frac{18}{3}[(10 + 0) + 4(25 + 38 + 22) + 2(30 + 36)]$

$\Rightarrow A = 6[(10) + 4(85) + 2(66)]$

$\Rightarrow A = 6[10 + 340 + 132]$

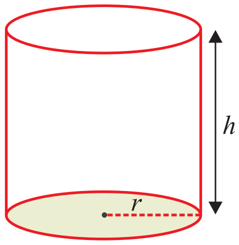
$\therefore A = 6[482] = 2892 \text{ m}^2$

1 (b) (ii)

$$\text{Volume} = \text{Area} \times \text{Depth} \Rightarrow \text{Depth} = \frac{\text{Volume}}{\text{Area}} = \frac{15000}{2892} = 5 \text{ m}$$

1 (c) (i)

CYLINDER



$V = \pi r^2 h$

Curved SA: $A = 2\pi rh$

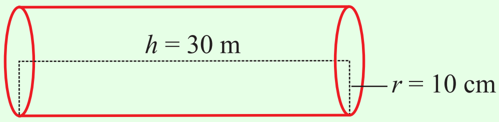
Total SA: $A = 2\pi rh + 2\pi r^2$ **14**

CYLINDRICAL ROD:

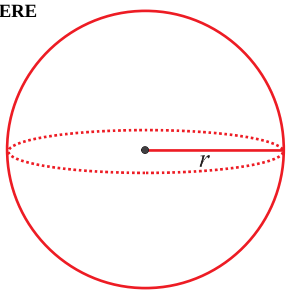
r = 10 cm,

h = 30 m = 3000 cm

$V = \pi r^2 h = \pi(10)^2(3000) = 300,000\pi \text{ cm}^3$



SPHERE



$V = \frac{4}{3}\pi r^3$

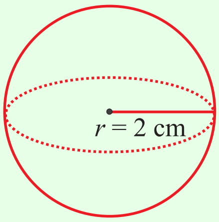
Curved SA: $A = 4\pi r^2$ **15**

1 (c) (ii)

SPHERE:

r = 2 cm

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2)^3 = \frac{32}{3}\pi \text{ cm}^3$



RECASTING: These are problems where solids of one type of shape are melted down and recast as solids in another shape. The volume of material in the original shape is the same as the volume in the new shape.

$$\text{Number of ball-bearings} = \frac{\text{Volume of rod}}{\text{Volume of sphere}} = \frac{300000\pi}{\frac{32}{3}\pi} = \frac{300000 \times 3}{32} = 28,125$$

1 (c) (iii)

If you divide the volume of the cylindrical rod by 225,000 you get the volume of the new sphere.

$$\text{Volume of new sphere } = \frac{300000\pi}{225000} = \frac{4}{3}\pi \text{ cm}^3$$

$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{4}{3}\pi = \frac{4}{3}\pi r^3$

$\Rightarrow 1 = r^3$

$\therefore r = 1 \text{ cm}$