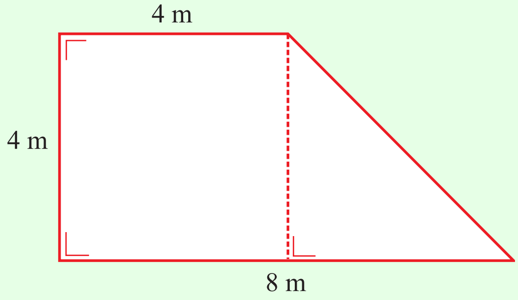
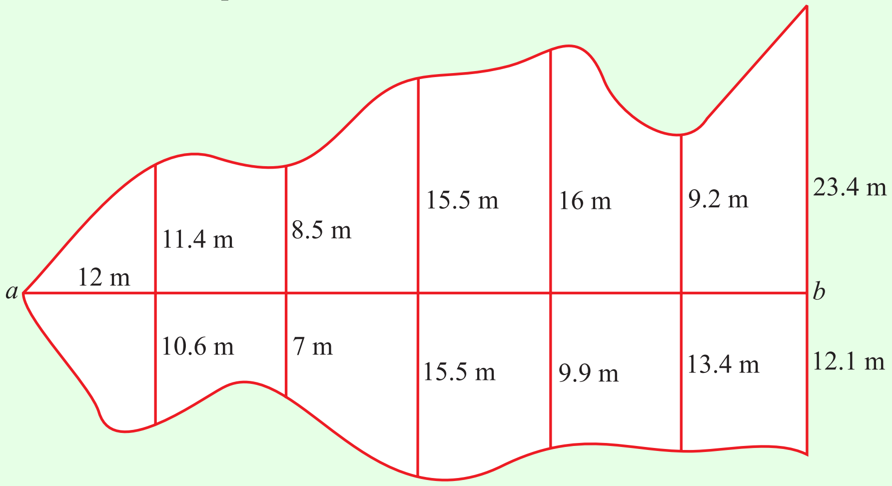
AREA & VOLUME (Q 1, PAPER 2)

2004

1 (a) Calculate the area of the figure in the diagram.



(b) The sketch shows a piece of land.



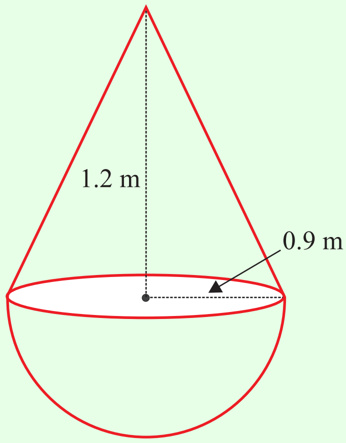
At equal intervals of 12 m along [*ab*], perpendicular measurements are made to the boundary, as shown on the sketch.
Use Simpson's Rule to estimate the area of the piece of land.

(c) A buoy at sea is in the shape of a hemisphere with a cone on top, as in the diagram.
The radius of the base of the cone is 0.9m and its vertical height is 1.2 m.

(i) Find the vertical height of the buoy.

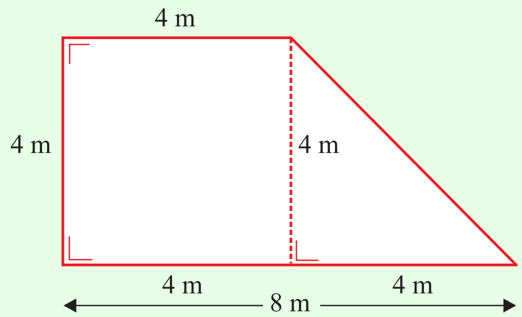
(ii) Find the volume of the buoy, in terms of π.

(iii) When the buoy floats, 0.8 m of its height is above water. Find, in terms of π, the volume of that part of the buoy that is above the water.

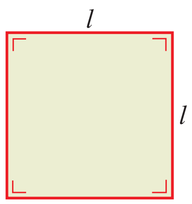


Solution

1 (a)



2. Square



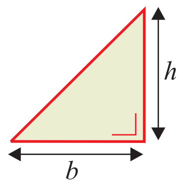
l: Length

$A = l \times l = l^2$
$P = 4l$ 2

Area of a right-angled triangle

You can find the area, A, by multiplying half the base, b, by the perpendicular height, h.

$A = \frac{1}{2}bh$ 4



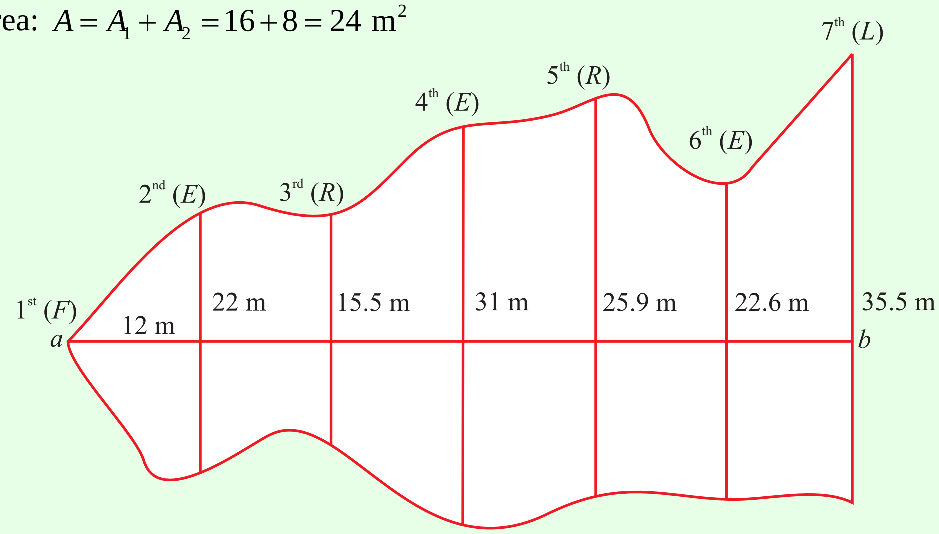
Area of figure = Area of square + Area of right-angled triangle

Area of square: $A_1 = l \times l = 4 \times 4 = 16 \text{ m}^2$

Area of right-angled triangle: $A_2 = \frac{1}{2}bh = \frac{1}{2}(4)(4) = 8 \text{ m}^2$

Total Area: $A = A_1 + A_2 = 16 + 8 = 24 \text{ m}^2$

1 (b)



$A \approx \frac{h}{3}$[(First + Last) + 4(Evens) + 2(Remaining Odds)] 11

$A \approx \frac{12}{3}[(0+35.5)+4(22+31+22.6)+2(15.5+25.9)]$

$\Rightarrow A \approx 4[(35.5)+4(75.6)+2(41.4)]$

$\Rightarrow A \approx 4[35.5+302.4+82.8]$

$\Rightarrow A \approx 4[420.7] = 1682.8 \text{ cm}^2$

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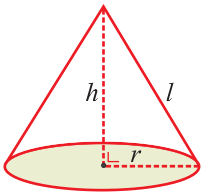
1 (c) (i)

CONE



$V = \frac{1}{3}\pi r^2 h$

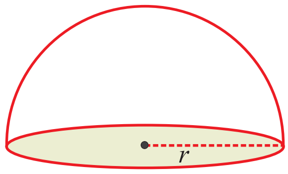
Curved SA: $A = \pi r l$

Total SA: $A = \pi r l + \pi r^2$

.... **17**

You can use Pythagoras on the cone: $l^2 = r^2 + h^2$

HEMISPHERE



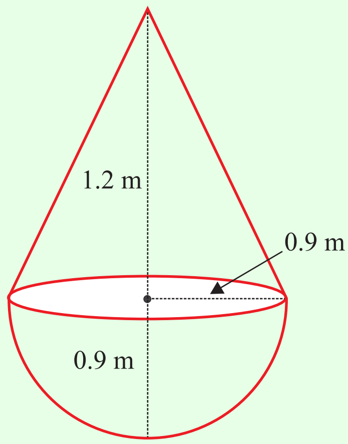
$V = \frac{2}{3}\pi r^3$

Curved SA: $A = 2\pi r^2$

Total SA: $A = 3\pi r^2$

.... **16**

As can be seen from the diagram, the vertical height of the buoy is 1.2 m + 0.9 m = 2.1 m



1 (c) (ii)

Volume of buoy = Volume of cone + Volume of hemisphere

CONE: r = 0.9 m, h = 1.2 m

$V_1 = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (0.9)^2 (1.2) = 0.324\pi \text{ cm}^3$

HEMISPHERE: r = 0.9 m

$V_2 = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi (0.9)^3 = 0.486\pi \text{ cm}^3$

BUOY: $V = V_1 + V_2 = 0.324\pi + 0.486\pi = 0.81\pi \text{ cm}^3$

1 (c) (iii)

A cone of height 0.8 m is above the water. What is its radius?
Compare the two similar triangles as shown. The ratio of their corresponding sides are equal.

$\therefore \frac{r_1}{h_1} = \frac{r_2}{h_2} \Rightarrow \frac{r_1}{0.8} = \frac{0.9}{1.2}$

$\Rightarrow r_1 = \frac{0.8 \times 0.9}{1.2} = 0.6 \text{ m}$

Volume of small cone:

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (0.6)^2 (0.8)$

$\therefore V = 0.096\pi \text{ m}^3$