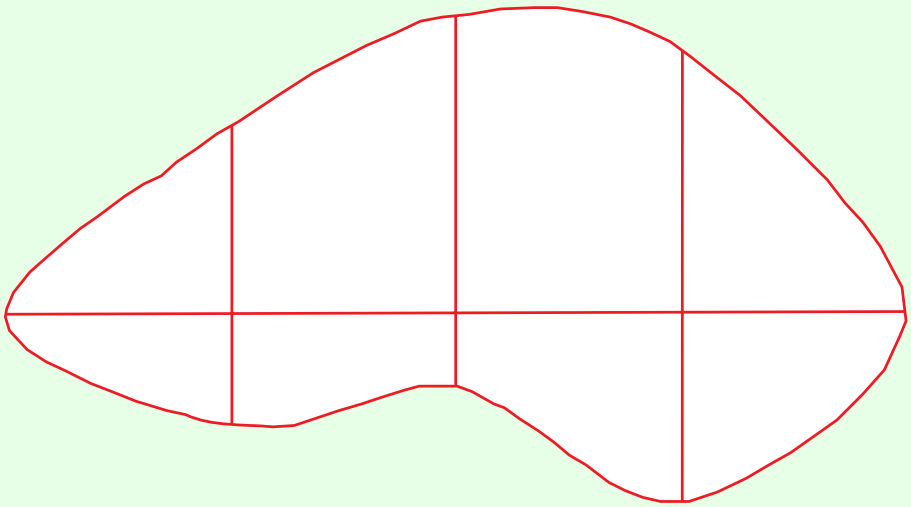
AREA & VOLUME (Q 1, PAPER 2)

2003

1 (a) A right-angled triangle has sides of length 8 cm, 15 cm and 17 cm.
Find its area.

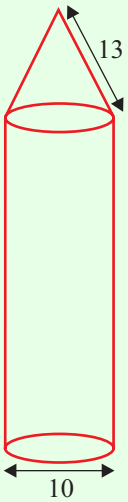
(b) In order to estimate the area of the irregular shape below, a horizontal line is drawn across the widest part of the shape and three offsets (perpendicular lines) are drawn at equal intervals along this line.



(i) Measure the horizontal line and the offsets as accurately as you can.
Make a rough sketch of the shape in your answerbook and record the measurements on it.

(ii) Use Simpson's Rule with these measurements to estimate the area of the shape.

(c) A wax crayon is in the shape of a cylinder of diameter 10 mm, surmounted by a cone of slant height 13 mm.



(i) Show that the vertical height of the cone is 12 mm.

(ii) Show that the volume of the cone is 100π mm^3.

(iii) Given that the volume of the cylinder is 15 times the volume of the cone, find the volume of the crayon, in cm^3, correct to two decimal places.

(iv) How many complete crayons like this one can be made from 1 kg of wax, given that each cm^3 of wax weighs 0.75 grammes?

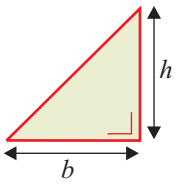
SOLUTION

1 (a)

AREA OF A RIGHT-ANGLED TRIANGLE

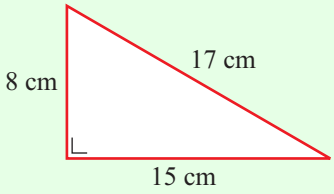
You can find the area, A, by multiplying half the base, b, by the perpendicular height, h.

$A = \frac{1}{2}bh$ 4

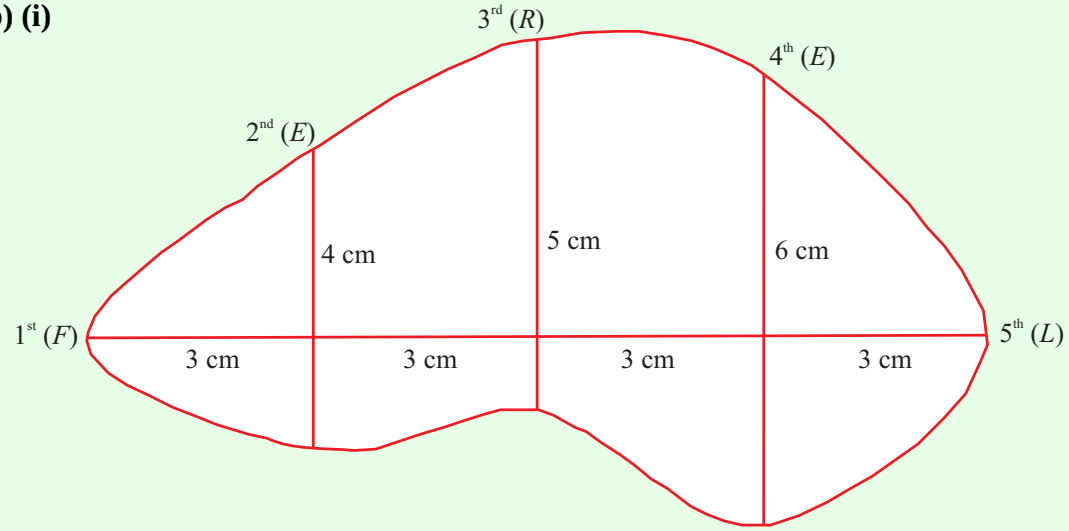


The longest side is the hypotenuse.

$A = \frac{1}{2}bh = \frac{1}{2}(15)(8) = 60 \text{ cm}^2$



1 (b) (i)



1 (b) (ii)

$A \approx \frac{h}{3}$ [(First + Last) + 4(Evens) + 2(Remaining Odds)] 11

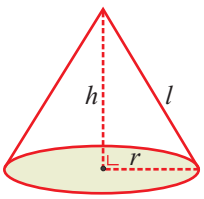
$A \approx \frac{3}{3}[(0+0)+4(4+6)+2(5)]$

$\Rightarrow A \approx 1[0+4(10)+10]$

$\therefore A \approx [40+10] = 50 \text{ cm}^2$

1 (c) (i)

CONE



$V = \frac{1}{3}\pi r^2 h$

Curved SA: $A = \pi rl$

Total SA: $A = \pi rl + \pi r^2$

..... 17

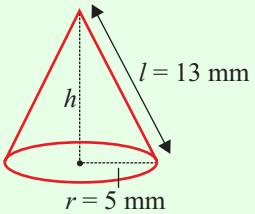
You can use Pythagoras on the cone: $l^2 = r^2 + h^2$

$l^2 = r^2 + h^2 \Rightarrow 13^2 = 5^2 + h^2$

$\Rightarrow h^2 = 169 - 25$

$\Rightarrow h^2 = 144$

$\therefore h = \sqrt{144} = 12 \text{ mm}$



1 (c) (ii)

$V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi(5)^2(12)$

$\therefore V = 100\pi \text{ mm}^3$

1 (c) (iii)

Volume of cylinder $= 15 \times 100\pi = 1500\pi \text{ mm}^3$

Volume of crayon = Volume of cone + Volume of cylinder $= 100\pi + 1500\pi = 1600\pi \text{ mm}^3$

$$1 \text{ cm} = 10 \text{ mm}$$
$$1 \text{ cm}^2 = 1 \text{ cm} \times 1 \text{ cm} = 10 \text{ mm} \times 10 \text{ mm} = 100 \text{ mm}^2$$
$$1 \text{ cm}^3 = 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} = 10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm} = 1000 \text{ mm}^3$$

$\therefore$ Volume of crayon $= 1.6\pi \text{ cm}^3 = 5.03 \text{ cm}^3$

1 (c) (iv)

Weight of one crayon $= 5.03 \times 0.75 \text{ g} = 3.7725 \text{ g}$

How many of these weights are contained in 1 kg (1000 g)?

Number of crayons $= \dfrac{1000 \text{ g}}{3.7725 \text{ g}} = 265$