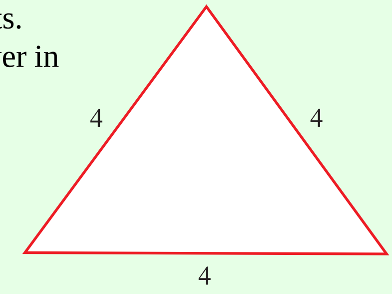
AREA & VOLUME (Q 1, PAPER 2)

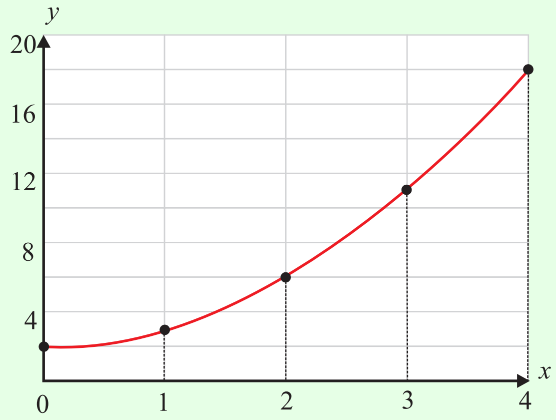
2002

1 (a) Each side of an equilateral triangle measures 4 units.
Calculate the area of the triangle, giving your answer in surd form.

Note: Area of a triangle $= \frac{1}{2}ab\sin C$.



(b) The diagram shows the curve $y = x^2 + 1$ in the domain $0 \leq x \leq 4$.



(i) Copy the following table. Then, complete it using the equation of the curve:

x	0	1	2	3	4
y					

(ii) Hence, use Simpson's Rule to estimate the area between the curve and the x-axis.

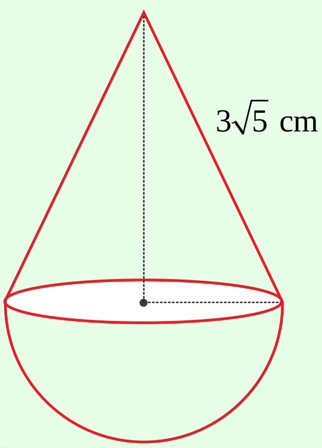
(c) A solid is in the shape of a hemisphere surmounted by a cone, as in the diagram.

(i) The volume of the hemisphere is 18π cm^3.
Find the radius of the hemisphere.

(ii) The slant height of the cone is $3\sqrt{5}$ cm.
Show that the vertical height of the cone is 6 cm.

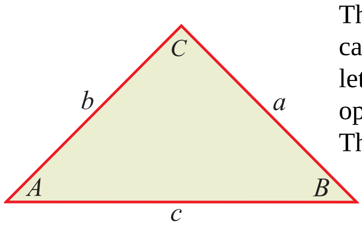
(iii) Show that the volume of the cone equals the volume of the hemisphere.

(iv) This solid is melted down and recast in the shape of a solid cylinder.
The height of the cylinder is 9 cm. Calculate its radius.



SOLUTION

1 (a)



The angles in triangles are represented by capital letters. Sides are represented by small letters. Side a is opposite angle A, side b is opposite angle B and so on.
The area, A, is given by the formula:

$A = \frac{1}{2}ab\sin C$ **5**

This formula is on page 6 of the tables.

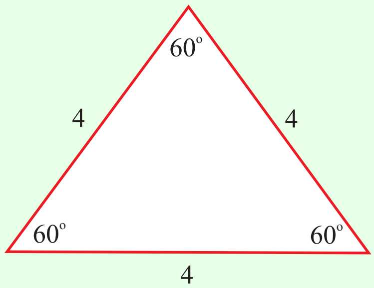
REMEMBER IT AS:

Area = $\frac{1}{2}\times$ Product of 2 sides $\times$ Sine of the included angle

Formula **5** can be written in three ways:

$$\text{Area } = \tfrac{1}{2}ab\sin C = \tfrac{1}{2}ac\sin B = \tfrac{1}{2}bc\sin A$$

All the angles are the same in an equilateral triangle.



$A = \frac{1}{2}(4)(4)\sin 60^\circ$

$\Rightarrow A = 8\sin 60^\circ = 8\times\frac{\sqrt{3}}{2}$

$\therefore A = 4\sqrt{3}$ square units

1 (b) (i)

$x = 0: y = (0)^2 + 1 = 0 + 1 = 1$

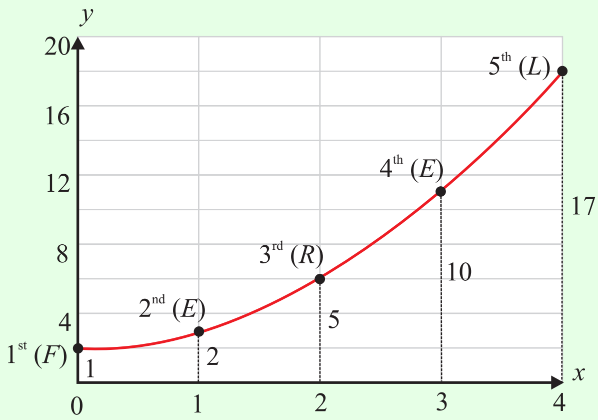
$x = 1: y = (1)^2 + 1 = 1 + 1 = 2$

$x = 2: y = (2)^2 + 1 = 4 + 1 = 5$

$x = 3: y = (3)^2 + 1 = 9 + 1 = 10$

$x = 4: y = (4)^2 + 1 = 16 + 1 = 17$

x	0	1	2	3	4
y	1	2	5	10	17



1 (b) (ii)

$A \approx \frac{h}{3}$[(First + Last) + 4(Evens) + 2(Remaining Odds)]...... **11**

$h = 1$ unit

$A \approx \frac{1}{3}[(1+17)+4(2+10)+2(5)]$

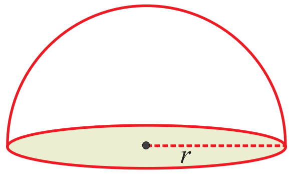
$\Rightarrow A \approx \frac{1}{3}[(18)+4(12)+2(5)]$

$\Rightarrow A \approx \frac{1}{3}[18+48+10]$

$\therefore A \approx \frac{1}{3}[76] = \frac{76}{3}$ units^2

1 (c) (i)

HEMISPHERE



$V = \frac{2}{3}\pi r^3$

Curved SA: $A = 2\pi r^2$

Total SA: $A = 3\pi r^2$ **16**

$V = 18\pi \text{ cm}^3$

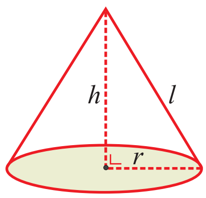
$V = \frac{2}{3}\pi r^3 \Rightarrow 18\pi = \frac{2}{3}\pi r^3$

$\Rightarrow r^3 = \dfrac{3\times 18}{2} = 27$

$\therefore r = \sqrt[3]{27} = 3 \text{ cm}$

1 (c) (ii)

CONE



$V = \frac{1}{3}\pi r^2 h$

Curved SA: $A = \pi r l$

Total SA: $A = \pi r l + \pi r^2$ **17**

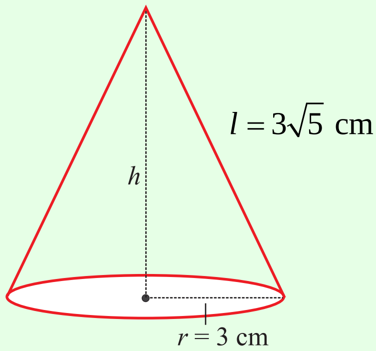
You can use Pythagoras on the cone: $l^2 = r^2 + h^2$

$l^2 = r^2 + h^2 \Rightarrow (3\sqrt{5})^2 = 3^2 + h^2$

$\Rightarrow 45 = 9 + h^2$

$\Rightarrow 36 = h^2$

$\therefore h = \sqrt{36} = 6 \text{ cm}$



1 (c) (iii)

Volume of cone: r = 3 cm, h = 6 cm

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (3)^2 (6) = 18\pi \text{ cm}^3$

Volume of hemisphere: r = 3 cm

$V = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi (3)^3 = 18\pi \text{ cm}^3$

1 (c) (iv)

RECASTING: These are problems where solids of one type of shape are melted down and recast as solids in another shape. The volume of material in the original shape is the same as the volume in the new shape.

Volume of solid = Volume of cone + Volume of hemisphere $= 18\pi + 18\pi = 36\pi \text{ cm}^3$

Cylinder: r = ?, h = 9 cm, $V = 36\pi \text{ cm}^3$

$V = \pi r^2 h \Rightarrow 36\pi = \pi r^2 (9)$

$\Rightarrow 36 = 9r^2 \Rightarrow r^2 = 4$

$\therefore r = \sqrt{4} = 2 \text{ cm}$