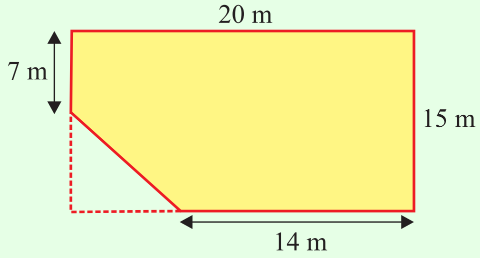
AREA & VOLUME (Q 1, PAPER 2)

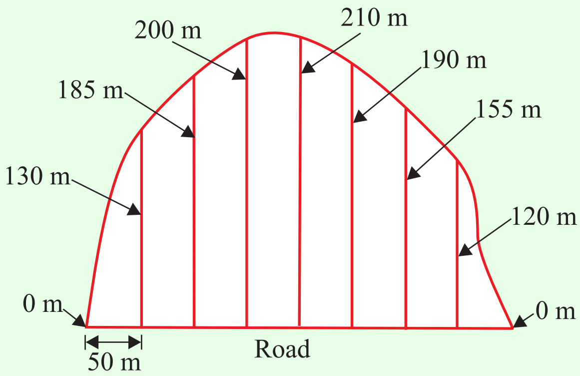
2000

1 (a) Calculate the area of the shaded region in the diagram.



(b) The sketch shows a piece of land covered by forest which lies on one side of a straight road.

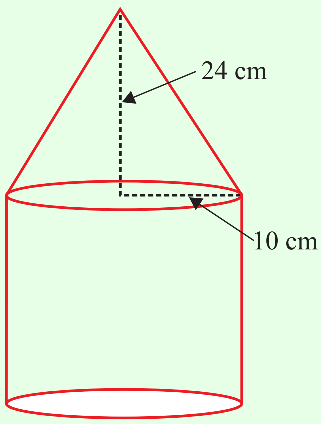
At equal intervals of 50 m along the road, perpendicular measurements of 130 m, 185 m, 200 m, 210 m, 190 m, 155 m and 120 m are made to the forest boundary.



Use Simpson's Rule to estimate the area of land covered by the forest. [See Tables, page 42.]

Give your answer in hectares. [Note: 1 hectare = 10 000 m^2.]

(c) A candle is in the shape of a cylinder surmounted by a cone, as in the diagram.



(i) The cone has height 24 cm and the length of the radius of its base is 10 cm. Find the volume of the cone in terms of π.

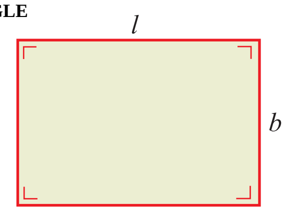
(ii) The height of the cylinder is equal to the slant height of the cone. Find the volume of the cylinder in terms of π.

(iii) A solid spherical ball of wax with radius of length r cm was used to make the candle. Calculate r, correct to one decimal place.

Solution

1 (a)

1. **Rectangle**

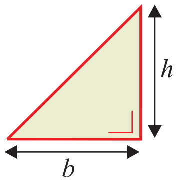


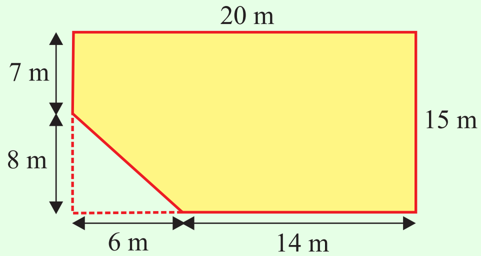
l: Length
b: Breadth

$A = l \times b$
$P = 2l + 2b = 2(l + b)$ **1**

Area of a right-angled triangle
You can find the area, A, by multiplying half the base, b, by the perpendicular height, h.

$A = \frac{1}{2}bh$ **4**





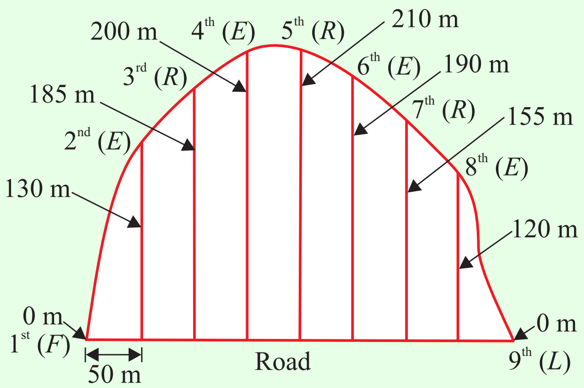
Shaded area (A) = Area of rectangle (A_1) – Area of right-angled triangle (A_2)

Area of rectangle: $A_1 = l \times b = 20 \times 15 = 300 \text{ m}^2$

Area of right-angled triangle: $A_2 = \frac{1}{2}bh = \frac{1}{2}(6)(8) = 24 \text{ m}^2$

Shaded Area: $A = A_1 - A_2 = 300 - 24 = 276 \text{ m}^2$

1 (b)



$A \approx \frac{h}{3}$ [(First + Last) + 4(Evens) + 2(Remaining Odds)]....... **11**

$h = 50$ m

$A \approx \frac{50}{3}[(0+0) + 4(130+200+190+120) + 2(185+210+155)]$

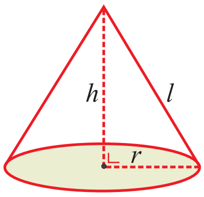
$\Rightarrow A \approx \frac{50}{3}[0 + 4(640) + 2(550)]$

$\Rightarrow A \approx \frac{50}{3}[2560 + 1100]$

$\therefore A \approx \frac{50}{3}[3660] = 61,000 \text{ m}^2 = 6.1$ hectares

1 (c) (i)

CONE



$V = \frac{1}{3}\pi r^2 h$

Curved SA: $A = \pi r l$

Total SA: $A = \pi r l + \pi r^2$

.... **17**

You can use Pythagoras on the cone: $l^2 = r^2 + h^2$

$h = 24$ cm, $r = 10$ cm

$V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi (10)^2 (24)$

$\therefore V = 800\pi \text{ cm}^3$

1 (c) (ii)

Find the slant height of cone.

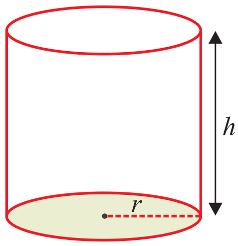
$l^2 = r^2 + h^2 \Rightarrow l^2 = 10^2 + 24^2$

$\Rightarrow l^2 = 100 + 576 = 676$

$\therefore l = \sqrt{676} = 26$ cm

Therefore, the height of the cylinder is 26 cm.

CYLINDER



$V = \pi r^2 h$

Curved SA: $A = 2\pi r h$

Total SA: $A = 2\pi r h + 2\pi r^2$

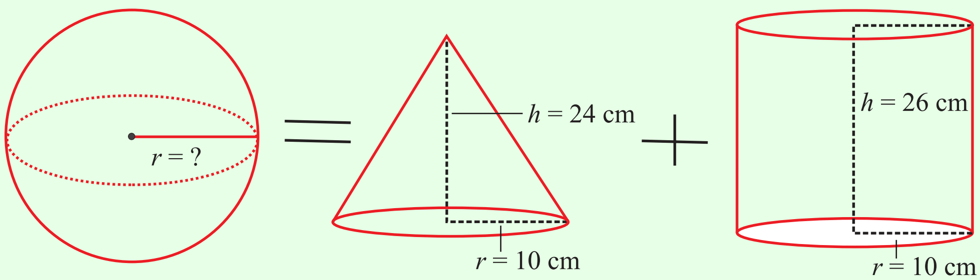
.... **14**

Cylinder: h = 26 cm, r = 10 cm

$V = \pi r^2 h \Rightarrow V = \pi (10)^2 (26)$

$\therefore V = 2600\pi \text{ cm}^3$

1 (c) (iii)



Volume of sphere = Volume of cone + Volume of cylinder $= 800\pi + 2600\pi = 3400\pi \text{ cm}^3$

Volume of sphere: $V = \frac{4}{3}\pi r^3$

$\therefore 3400\pi = \frac{4}{3}\pi r^3 \Rightarrow \dfrac{3 \times 3400}{4} = r^3$

$\Rightarrow r^3 = 2550$

$\therefore r = \sqrt[3]{2550} = 13.7$ cm