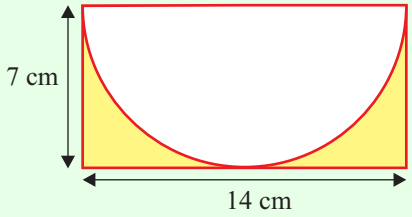
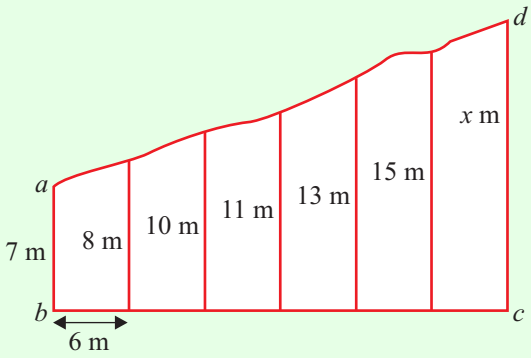
Area & Volume (Q 1, Paper 2)

1998

1 (a) A rectangular piece of metal measures 7 cm by 14 cm.
A semi-circular section with radius of length 7 cm is removed.
Calculate the area of the remaining piece of metal.
Take $\pi = \frac{22}{7}$.



(b) The sketch shows a field *abcd* which has one uneven edge. At equal intervals of 6 m along [*bc*] perpendicular measurements of 7 m, 8 m, 10 m, 11 m, 13 m, 15 m and *x* m are made to the top of the field.



Using Simpson's Rule the area of the field is calculated to be 410 m^2.
Calculate the value of *x*. [See Tables, page 42.]

(c) Find the volume of a solid sphere with a diameter of length 3 cm. Give your answer in terms of π.

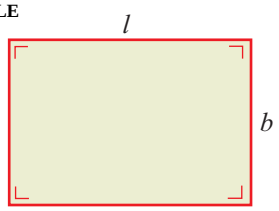
A cylindrical vessel with internal diameter of length 15 cm contains water. The surface of the water is 11 cm from the top of the vessel.

How many solid spheres, each with diameter of length 3 cm, must be placed in the vessel in order to bring the surface of the water to 1 cm from the top of the vessel?
Assume that all the spheres are submerged in the water.

SOLUTION

1 (a)

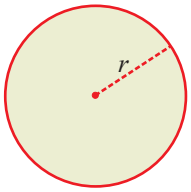
1. **RECTANGLE**



l: Length
b: Breadth

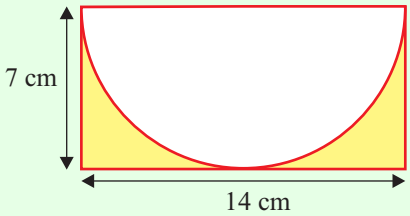
$$A = l \times b$$
$$P = 2l + 2b = 2(l + b) \quad \ldots\ldots \text{(1)}$$

6. **CIRCLE**



L: Length of Circumference
r: Radius

$$L = 2\pi r \quad \ldots\ldots \text{(7)}$$
$$A = \pi r^2 \quad \ldots\ldots \text{(8)}$$



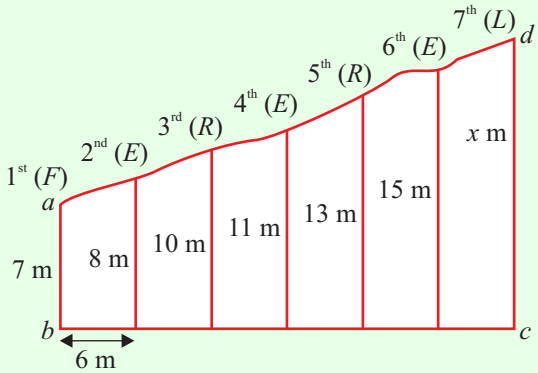
Shaded area (A) = Area of rectangle (A_1) – Area of half the circle (A_2)

Area of rectangle: $A_1 = l \times b = 14 \times 7 = 98 \text{ cm}^2$

Area of half circle: $A_2 = \frac{1}{2}\pi r^2 = \frac{1}{2}(\frac{22}{7})(7)^2 = 77 \text{ cm}^2$

Shaded area: $A = A_1 - A_2 = 98 - 77 = 21 \text{ cm}^2$

1 (b)



$$A \approx \tfrac{h}{3}\,[(\text{First} + \text{Last}) + 4(\text{Evens}) + 2(\text{Remaining Odds})] \quad \ldots\ldots \text{(11)}$$

$h = 6 \text{ m}, \; A = 410 \text{ m}^2$

$410 = \frac{6}{3}[(7 + x) + 4(8 + 11 + 15) + 2(10 + 13)]$

$\Rightarrow 410 = 2[(7 + x) + 4(34) + 2(23)]$

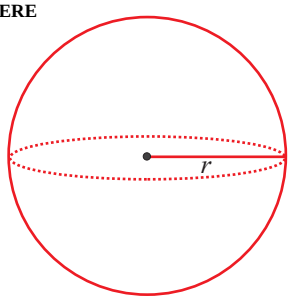
$\Rightarrow 410 = 2[7 + x + 136 + 46]$

$\Rightarrow 205 = [x + 189]$

$\therefore x = 205 - 189 = 16 \text{ m}$

1 (c)

SPHERE



$V = \frac{4}{3}\pi r^3$

Curved SA: $A = 4\pi r^2$ 15

Sphere:

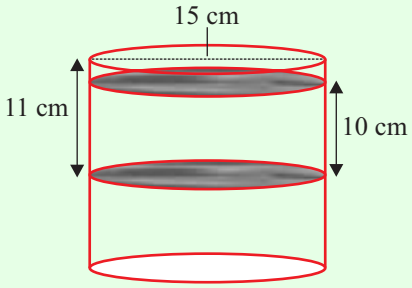
$r = \frac{3}{2}$ cm

$V = \frac{4}{3}\pi r^3 \Rightarrow V = \frac{4}{3}\pi(\frac{3}{2})^3$

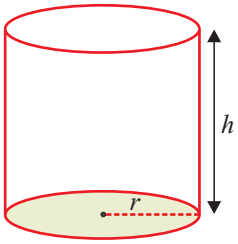
$\Rightarrow V = \frac{4}{3}\pi(\frac{27}{8})$

$\therefore V = \frac{9}{2}\pi$ cm^3

When the spheres are submerged in the water they raise the height of the water. The spheres displace of volume of water in the cylinder of height 10 cm. Find the volume of a cylinder of height 10 cm.



CYLINDER



$V = \pi r^2 h$

Curved SA: $A = 2\pi rh$

Total SA: $A = 2\pi rh + 2\pi r^2$ 14

CYLINDER:

$r = \frac{15}{2}$ cm, $h = 10$ cm

$V = \pi r^2 h \Rightarrow V = \pi(\frac{15}{2})^2(10)$

$\therefore V = \frac{1125}{2}\pi$ cm^3

Divide the volume of a sphere into this volume to find the number of spheres submerged.

Number of spheres $= \dfrac{\frac{1125}{2}\pi}{\frac{9}{2}\pi} = 125$