## Area \& Volume (Q 1, Paper 2)

## 1997

1 (a) Find the slant height, $l$, of a cone which has perpendicular height of 4 cm and base with radius of length 3 cm .
Write down the curved surface area of the cone in terms of $\pi$.

(b) The diagram shows a sketch of a piece of paper abcd with one uneven edge. At equal intervals of $h \mathrm{~cm}$ along [ $b c$ ], perpendicular measurements of $12 \mathrm{~cm}, 8 \mathrm{~cm}, 9 \mathrm{~cm}$, $6 \mathrm{~cm}, 5 \mathrm{~cm}, 7 \mathrm{~cm}$ and 11 cm are made to the top edge.


Use Simpson's Rule the area of the piece of paper is estimated to be $180 \mathrm{~cm}^{2}$. Calculate the value of $h$. [See Tables, page 42.]
(c) Find the volume of a solid sphere which has radius of length 2.1 cm . Give your answer correct to the nearest $\mathrm{cm}^{3}$. Take $\frac{22}{7}$ as an approximation of $\pi$.

This sphere and a solid cube with edge of length 3 cm are completely submerged in water in a cylinder. The cylinder has radius of length $r \mathrm{~cm}$.

Both the sphere and the cube are then removed from the cylinder. The water level drops by 4 cm . Find $r$, correct to one place of decimals. [Take $\pi=\frac{22}{7}$.]

## Solution

1 (a)
Cone


$$
V=\frac{1}{3} \pi r^{2} h
$$

Curved SA: $A=\pi r l$
Total SA: $A=\pi r l+\pi r^{2}$
You can use Pythagoras on the cone: $l^{2}=r^{2}+h^{2}$
$h=4 \mathrm{~cm}, r=3 \mathrm{~cm}$
$l^{2}=r^{2}+h^{2} \Rightarrow l^{2}=3^{2}+4^{2}$
$\Rightarrow l^{2}=9+16=25$
$\therefore l=\sqrt{25}=5 \mathrm{~cm}$
Curved Surface Area:
$A=\pi r l \Rightarrow A=\pi(3)(5)$
$\therefore A=15 \pi \mathrm{~cm}^{2}$
1 (b)

$A \approx \frac{h}{3}[($ First + Last $)+4$ (Evens $)+2($ Remaining Odds $\left.)\right]$
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$A=180 \mathrm{~cm}^{2}$
$180=\frac{h}{3}[(12+11)+4(8+6+7)+2(9+5)]$
$\Rightarrow 180=\frac{h}{3}[(23)+4(21)+2(14)]$
$\Rightarrow 180=\frac{h}{3}[23+84+28]$
$\Rightarrow 180=\frac{h}{3}[135]$
$\Rightarrow 180=h[45]$
$\therefore h=\frac{180}{45}=4 \mathrm{~cm}$

1 (c)

$r=2.1 \mathrm{~cm}, \pi=\frac{22}{7}$
$V=\frac{4}{3} \pi r^{3} \Rightarrow V=\frac{4}{3}\left(\frac{22}{7}\right)(2.1)^{3}$
$\therefore V=39 \mathrm{~cm}^{3}$


## Cylinder



$$
V=\pi r^{2} h
$$

Curved SA: $A=2 \pi r h$
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Total SA: $A=2 \pi r h+2 \pi r^{2}$

Firstly, find the total volume of the sphere and cube.
Cube: $l=3 \mathrm{~cm}$
$V=l^{3} \Rightarrow V=(3)^{3}=27 \mathrm{~cm}^{3}$
Total volume of sphere and cube $=39+27=66 \mathrm{~cm}^{3}$
When the sphere and cube are removed from a cylinder of water, the height $h$ falls by 4 cm .
The volume of this cylinder of water equals the volume of the sphere and cube.
$r=$ ?, $h=4 \mathrm{~cm}, V=66 \mathrm{~cm}^{3}$
$V=\pi r^{2} h \Rightarrow 66=\left(\frac{22}{7}\right) r^{2}(4)$
$\Rightarrow r^{2}=\frac{66 \times 7}{22 \times 4}=\frac{21}{4}$
$\therefore r=\sqrt{\frac{21}{4}}=2.3 \mathrm{~cm}$

