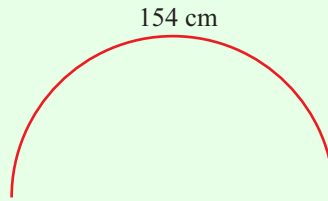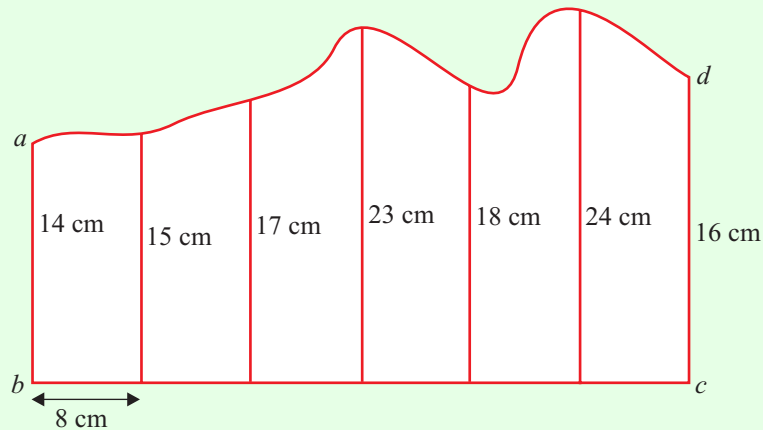## AREA & VOLUME (Q 1, PAPER 2)

# 1996

1  (a)  A piece of wire of length 154 cm
      is in the shape of a semicircle.
      Find the radius length of the semicircle.
      Take $\pi = \frac{22}{7}$.

      154 cm

   (b)  A sketch to estimate the area of a building site abcd is shown. At intervals of 8 m
       along [bc], perpendicular measurements of 14 m, 15 m, 17 m, 23 m, 18 m, 24 m
       and 16 m are made to the top boundary.

       a

       14 cm    15 cm    17 cm    23 cm    18 cm    24 cm    16 cm

       b                                                              c
       8 cm

       Use Simpson's Rule to estimate the area of the building site.
       [See Tables, page 42].

   (c)  A solid cylinder, made of lead, has a radius of length 15 cm and height of 135 cm.
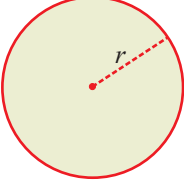       Find its volume in terms of $\pi$.

       The solid cylinder is melted down and recast to make four identical right circular
       solid cones. The height of each cone is equal to twice the length of its base radius.

       Calculate the base radius length of the cones.

**SOLUTION**

**1 (a)**

> **6. CIRCLE**
>
> *L*: Length of Circumference
> *r*: Radius
>
> $L = 2\pi r$ ....... **7**
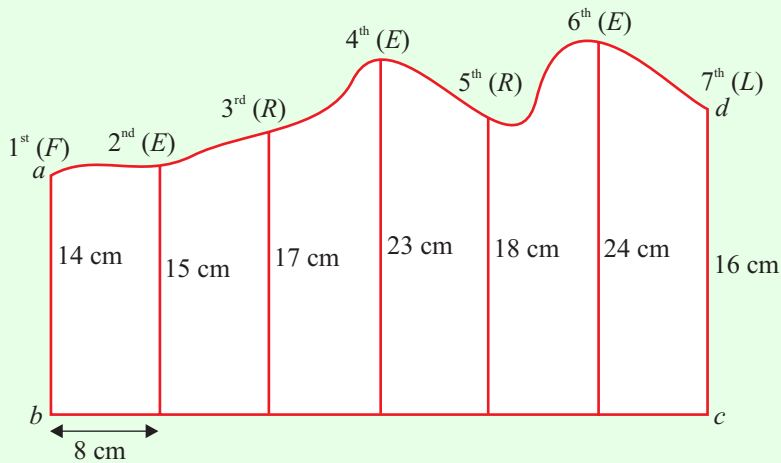>
> $A = \pi r^2$ ....... **8**

The length of the circumference of a semicircle is given by the formula $\pi r$.

**SEMICIRCLE:** $L = 154$ cm, $\pi = \frac{22}{7}$

$L = \pi r \Rightarrow 154 = \frac{22}{7} r$

$\therefore r = \dfrac{154 \times 7}{22} = 49$ cm

**1 (b)**

$$A \approx \tfrac{h}{3} \left[ (\text{First} + \text{Last}) + 4(\text{Evens}) + 2(\text{Remaining Odds}) \right] \dots \text{(11)}$$

$h = 8$ cm
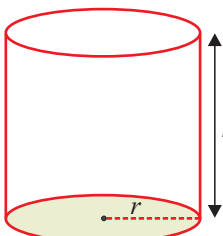
$A \approx \frac{8}{3}[(14+16) + 4(15+23+24) + 2(17+18)]$

$\Rightarrow A \approx \frac{8}{3}[(30) + 4(62) + 2(35)]$

$\Rightarrow A \approx \frac{8}{3}[30 + 248 + 70]$

$\therefore A \approx \frac{8}{3}[348] = 928$ cm$^3$

**1 (c)**

> **CYLINDER**
>
> $V = \pi r^2 h$
>
> Curved SA: $A = 2\pi rh$ ....... **14**
>
> Total SA: $A = 2\pi rh + 2\pi r^2$
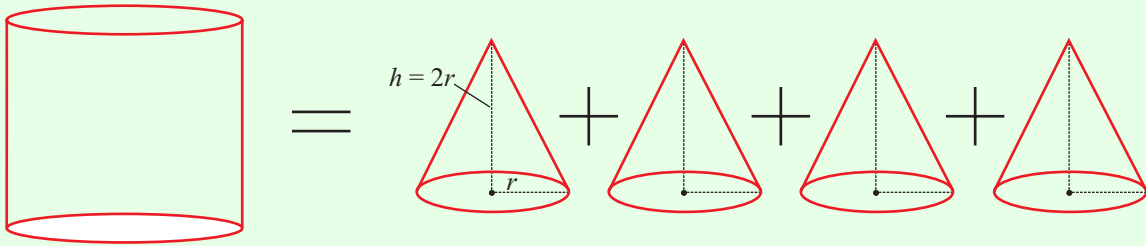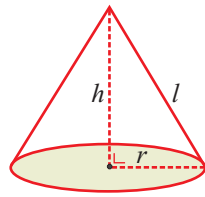
**CYLINDER:** $r = 15$ cm, $h = 135$ cm

$V = \pi r^2 h \Rightarrow V = \pi (15)^2 (135)$

$\therefore V = 30375\pi$ cm$^3$

**RECASTING**: These are problems where solids of one type of shape are melted down and recast as solids in another shape. The volume of material in the original shape is the same as the volume in the new shape.

$h = 2r$

**CONE**

$$V = \tfrac{1}{3}\pi r^2 h$$

Curved SA: $A = \pi r l$

Total SA: $A = \pi r l + \pi r^2$

....... **17**

You can use Pythagoras on the cone: $l^2 = r^2 + h^2$

**CONE**: $r = r$, $h = 2r$

$$V = \tfrac{1}{3}\pi r^2 h \Rightarrow V = \tfrac{1}{3}\pi r^2 (2r)$$

$$\therefore V = \tfrac{2}{3}\pi r^3$$

Volume of cylinder = 4 times the volume of the cone

$$\therefore 30375\pi = 4 \times \tfrac{2}{3}\pi r^3$$

$$\Rightarrow \frac{30375 \times 3}{8} = r^3$$

$$\therefore r = \sqrt[3]{\frac{30375 \times 3}{8}} = 22.5 \text{ cm}$$