ALGEBRA (Q 2 & 3, PAPER 1)

LESSON NO. 7: SIMULTANEOUS EQUATIONS

2007

3 (b) Solve the simultaneous equations

$$\frac{x}{4}-\frac{y}{3}=\frac{5}{6}$$
$$2x-6=3y.$$

SOLUTION

STEPS
1. Work on each equation so that you have the x and y terms on the left-hand side and the number on the other side.
2. Eliminate either the x's or the y's.
3. Solve for the remaining letter.
4. Substitute into either of the original equations to get the other letter.

1. $\frac{x}{4}-\frac{y}{3}=\frac{5}{6}(\times 12)\Rightarrow 3x-4y=10....(\mathbf{1})$

$2x-6=3y\Rightarrow 2x-3y=6...(\mathbf{2})$

2. Eliminate the x's:

$$\begin{aligned}&3x-4y=10....(\mathbf{1})(\times 2)\\&2x-3y=6.....(\mathbf{2})(\times -3)\end{aligned}\quad\rightarrow\quad\begin{aligned}&6x-8y=20\\&-6x+9y=-18\\&\hline\\&y=2\end{aligned}$$

3. $y=2$

4. Substitute into equation (1): $2x-3y=6\Rightarrow 2x-3(2)=6$

$\Rightarrow 2x-6=6\Rightarrow 2x=12$

$\therefore x=6$

2006

3 (b) Solve for x and y

$$x - 2y = 10$$
$$x^2 + y^2 = 20.$$

SOLUTION

STEPS
1. Eliminate a letter from the linear equation.
2. Substitute into quadratic and solve for the other letter.
3. Substitute these values into the linear to get all solutions.

1. Isolate the x as this is the easier letter to get on its own.

$$x - 2y = 10$$
$$\Rightarrow x = 2y + 10 ... (\mathbf{A})$$

→

2.

$$x^2 + y^2 = 20$$
$$\Rightarrow (2y+10)^2 + y^2 = 20$$
$$\Rightarrow 4y^2 + 40y + 100 + y^2 = 20$$
$$\Rightarrow 5y^2 + 40y + 80 = 0$$
$$\Rightarrow y^2 + 8y + 16 = 0$$
$$\Rightarrow (y+4)(y+4) = 0$$
$$\therefore y = -4$$

←

3. Substitute this value of y into Eqn. (**A**) to obtain the x value.

$$y = -4: x = 2(-4) + 10 = 2$$

ANSWER: $(2, -4)$

2005

2 (b) Solve for x and y

$$x + 3 = 2y$$
$$xy - 7y + 8 = 0.$$

SOLUTION

STEPS
1. Eliminate a letter from the linear equation.
2. Substitute into quadratic and solve for the other letter.
3. Substitute these values into the linear to get all solutions.

1. Isolate the x as this is the easier letter to get on its own.

$$x + 3 = 2y$$
$$\Rightarrow x = 2y - 3 ....\mathbf{(A)}$$

→

2.

$$xy - 7y + 8 = 0$$
$$\Rightarrow (2y-3)y - 7y + 8 = 0$$
$$\Rightarrow 2y^2 - 3y - 7y + 8 = 0$$
$$\Rightarrow 2y^2 - 10y + 8 = 0$$
$$\Rightarrow y^2 - 5y + 4 = 0$$
$$\Rightarrow (y-1)(y-4) = 0$$
$$\Rightarrow y = 1,\ y = 4$$

←

3. Substitute the values of y into Eqn. (**A**) to obtain the x values.

$$y = 1:\ x = 2(1) - 3 = 2 - 3 = -1$$
$$y = 4:\ x = 2(4) - 3 = 8 - 3 = 5$$

ANSWER: $(-1, 1), (5, 4)$

2004

3 (b) Solve for x and y

$$x + y = 1$$
$$x^2 + y^2 = 13.$$

SOLUTION

STEPS
1. Eliminate a letter from the linear equation.
2. Substitute into quadratic and solve for the other letter.
3. Substitute these values into the linear to get all solutions.

1. Isolate the x.

$$x + y = 1$$
$$\Rightarrow x = 1 - y \ldots (\mathbf{A})$$

→

2.

$$x^2 + y^2 = 1$$
$$\Rightarrow (1-y)^2 + y^2 = 13$$
$$\Rightarrow 1 - 2y + y^2 + y^2 = 13$$
$$\Rightarrow 2y^2 - 2y - 12 = 0$$
$$\Rightarrow y^2 - y - 6 = 0$$
$$\Rightarrow (y-3)(y+2) = 0$$
$$\therefore y = -2,\ 3$$

3. Substitute the values of y into Eqn. (**A**) to obtain the x values.

←

$$y = -2: x = 1 - (-2) = 3$$
$$y = 3: x = 1 - (3) = 2$$

ANSWER: $(3, -2), (-2, 3)$

2002

3 (b) (i) Solve for x and y

$$y = 10 - 2x$$
$$x^2 + y^2 = 25.$$

(ii) Hence, find the two possible values of $x^3 + y^3$.

SOLUTION

STEPS
1. Eliminate a letter from the linear equation.
2. Substitute into quadratic and solve for the other letter.
3. Substitute these values into the linear to get all solutions.

1.

$$y = 10 - 2x \ldots (\mathbf{A})$$

→

2.

$$x^2 + y^2 = 25$$
$$\Rightarrow x^2 + (10 - 2x)^2 = 25$$
$$\Rightarrow x^2 + 100 - 40x + 4x^2 = 25$$
$$\Rightarrow 5x^2 - 40x + 75 = 0$$
$$\Rightarrow x^2 - 8x + 15 = 0$$
$$\Rightarrow (x-5)(x-3) = 0$$
$$\Rightarrow x = 3,\ 5$$

3. Substitute the values of x into Eqn. (**A**) to obtain the y values.

←

$$x = 3:\ y = 10 - 2(3) = 4$$
$$x = 5:\ y = 10 - 2(5) = 0$$

ANSWER: $(3, 4), (5, 0)$

3 (b) (ii)

$x = 3, y = 4$: $\therefore x^3 + y^3 = 3^3 + 4^3 = 27 + 64 = 91$

$x = 5, y = 0$: $\therefore x^3 + y^3 = 5^3 + 0^3 = 125 + 0 = 125$

2001

2 (b) Solve for x and y

$$x+2y=3$$
$$x^2-y^2=24.$$

SOLUTION

STEPS
1. Eliminate a letter from the linear equation.
2. Substitute into quadratic and solve for the other letter.
3. Substitute these values into the linear to get all solutions.

1. Isolate the x as this is the easier letter to get on its own.

$$x+2y=3$$
$$\Rightarrow x=3-2y....(\mathbf{A})$$

2.

$$x^2-y^2=24$$
$$\Rightarrow (3-2y)^2-y^2=24$$
$$\Rightarrow 9-12y+4y^2-y^2=24$$
$$\Rightarrow 3y^2-12y-15=0$$
$$\Rightarrow y^2-4y-5=0$$
$$\Rightarrow (y-5)(y+1)=0$$
$$\Rightarrow y=5,\ -1$$

3. Substitute this value of y into Eqn. (**A**) to obtain the x value.

$$y=5: x=3-2(5)=-7$$
$$y=-1: x=3-2(-1)=5$$

ANSWER: $(-7, 5), (5, -1)$

2000

2 (b) Solve for x and y

$$x-3y=1$$
$$x^2-y^2=0.$$

SOLUTION

STEPS
1. Eliminate a letter from the linear equation.
2. Substitute into quadratic and solve for the other letter.
3. Substitute these values into the linear to get all solutions.

1. Isolate the x as this is the easier letter to get on its own.

$$x-3y=1$$
$$\Rightarrow x=3y+1...(\mathbf{A})$$

2.

$$\Rightarrow x^2-y^2=0$$
$$\Rightarrow (3y+1)^2-y^2=0$$
$$\Rightarrow 9y^2+6y+1-y^2=0$$
$$\Rightarrow 8y^2+6y+1=0$$
$$\Rightarrow (4y+1)(2y+1)=0$$
$$\therefore y=-\tfrac{1}{4},\ -\tfrac{1}{2}$$

3. Substitute this value of y into Eqn. (**A**) to obtain the x value.

$$y=-\tfrac{1}{4}: x=3(-\tfrac{1}{4})+1=-\tfrac{3}{4}+1=\tfrac{1}{4}$$
$$y=-\tfrac{1}{2}: x=3(-\tfrac{1}{2})+1=-\tfrac{3}{2}+1=-\tfrac{1}{2}$$

ANSWER: $(\tfrac{1}{4}, -\tfrac{1}{4}), (-\tfrac{1}{2}, -\tfrac{1}{2})$

1999

3 (b) Solve for x and y

$$x+2y=6$$
$$x^2+y^2=17.$$

SOLUTION

STEPS
1. Eliminate a letter from the linear equation.
2. Substitute into quadratic and solve for the other letter.
3. Substitute these values into the linear to get all solutions.

1. Isolate the x as this is the easier letter to get on its own.

$$x+2y=6$$
$$\Rightarrow x=6-2y....(\mathbf{A})$$

2.

$$x^2+y^2=17$$
$$\Rightarrow (6-2y)^2+y^2=17$$
$$\Rightarrow 36-24y+4y^2+y^2=17$$
$$\Rightarrow 5y^2-24y+19=0$$
$$\Rightarrow (5y-19)(y-1)=0$$
$$\therefore y=\tfrac{19}{5},\ 1$$

3. Substitute this value of y into Eqn. (**A**) to obtain the x value.

$$y=1: x=6-2(1)=6-2=4$$
$$y=\tfrac{19}{5}: x=6-2(\tfrac{19}{5})=6-\tfrac{38}{5}=-\tfrac{8}{5}$$

ANSWER: $(-\frac{8}{5}, \frac{19}{5})$, (4, 1)

1998

2 (a) Solve

$$5x-2y=13$$
$$3(x-4)=4y.$$

SOLUTION

STEPS
1. Work on each equation so that you have the x and y terms on the left-hand side and the number on the other side.
2. Eliminate either the x's or the y's.
3. Solve for the remaining letter.
4. Substitute into either of the original equations to get the other letter.

$3(x-4)=4y \Rightarrow 3x-12=4y \Rightarrow 3x-4y=12$

1. Eliminate the y's by multiplying Eqn. **(A)** by -2.

$$5x-2y=13...(\mathbf{A})(\times -2)$$
$$3x-4y=12...(\mathbf{B})$$

2. Add the two equations together.

$$-10x+4y=-26$$
$$3x-4y=12$$
$$-7x=-14$$

3. Solve the resulting equation for x.

$$-7x=-14 \Rightarrow x=2$$

4. Substitute the value of x into Eqn. **(B)**

$$3(2)-4y=12$$
$$\Rightarrow 6-4y=12$$
$$\Rightarrow -4y=6 \Rightarrow y=-\tfrac{3}{2}$$

ANSWER: $x=2,\ y=-\frac{3}{2}$

1996

2 (a) Solve

$$2x - y = 7$$
$$x + 2y = 6.$$

SOLUTION

STEPS
1. Work on each equation so that you have the x and y terms on the left-hand side and the number on the other side.
2. Eliminate either the x's or the y's.
3. Solve for the remaining letter.
4. Substitute into either of the original equations to get the other letter.

1. Eliminate the y's by multiplying Eqn. **(A)** by 2.

$$2x - y = 7 \ldots (\mathbf{A})(\times 2)$$
$$x + 2y = 6 \ldots (\mathbf{B})$$

2. Add the two equations together.

$$4x - 2y = 14$$
$$x + 2y = 6$$
$$5x \quad = 20$$

3. Solve the resulting equation for x.

$$5x = 20 \Rightarrow x = 4$$

4. Substitute the value of x into Eqn. **(B)**

$$(4) + 2y = 6$$
$$\Rightarrow 2y = 2$$
$$\Rightarrow y = 1$$

ANSWER: $x = 4$, $y = 1$