ALGEBRA (Q 2 & 3, PAPER 1)

LESSON NO. 5: INDEX EQUATIONS

2007

2 (b) (ii) Find the value of x for which $2^{x+3} = 4^x$.

SOLUTION

> **STEPS**
> **1.** Tidy up both sides using the Power Rules until you have the same base and nothing else on both sides.
> **2.** Put the powers equal to one another.
> **3.** Solve for the variable (usually x.)

1. $2^{x+3} = 4^x \Rightarrow 2^{x+3} = (2^2)^x \Rightarrow 2^{x+3} = 2^{2x}$

2. $x+3 = 2x$

3. $3 = 2x - x \Rightarrow x = 3$

> **POWER RULES**
> 5. $(a^m)^n = a^{mn}$ **Ex.** $(x^3)^2 = x^6$

2004

2 (c) (i) Evaluate $8^{\frac{1}{3}}$.

(ii) Express $4^{\frac{1}{4}}$ in the form 2^k, $k \in \mathbf{Q}$.

(iii) Solve for x the equation

$$(8^{\frac{1}{3}})(4^{\frac{1}{4}}) = 2^{5-x}.$$

SOLUTION

2 (c) (i)

$8^{\frac{1}{3}} = 2$

> The third root of 8 is the number multiplied by itself three times to give 8.

2 (c) (ii)

$4^{\frac{1}{4}} = (2^2)^{\frac{1}{4}} = 2^{\frac{1}{2}}$

> **POWER RULES**
> 5. $(a^m)^n = a^{mn}$

2 (c) (iii)

> **STEPS**
> **1.** Tidy up both sides using the Power Rules until you have the same base and nothing else on both sides.
> **2.** Put the powers equal to one another.
> **3.** Solve for the variable (usually x.)

1. $(8^{\frac{1}{3}})(4^{\frac{1}{4}}) = 2^{5-x} \Rightarrow (2^1)(2^{\frac{1}{2}}) = 2^{5-x}$

$\Rightarrow 2^{\frac{3}{2}} = 2^{5-x}$

> **POWER RULES**
> 1. $a^m \times a^n = a^{m+n}$

2. $\therefore \frac{3}{2} = 5 - x$

3. $\frac{3}{2} = 5 - x \Rightarrow 3 = 10 - 2x$ [Multiplied across by 2.]

$\Rightarrow 2x = 10 - 3 \Rightarrow 2x = 7$

$\therefore x = \frac{7}{2}$

2003

2 (b) (i) Evaluate $9^{\frac{1}{2}}$.

(ii) Express $\sqrt{8}$ in the form 2^k, $k \in \mathbf{Q}$.

(iii) Solve for x the equation $25^x = 5^{6-x}$.

SOLUTION

2 (b) (i)

$9^{\frac{1}{2}} = 3$

What number multiplied by itself gives 9?

2 (b) (ii)

$\sqrt{8} = 8^{\frac{1}{2}} = (2^3)^{\frac{1}{2}} = 2^{\frac{3}{2}}$

POWER RULES

5. $(a^m)^n = a^{mn}$ **Ex.** $(x^3)^2 = x^6$

6. $\sqrt{a} = a^{\frac{1}{2}}$ **Ex.** $\sqrt{9} = 9^{\frac{1}{2}} = 3$

2 (b) (iii)

STEPS

1. Tidy up both sides using the Power Rules until you have the same base and nothing else on both sides.
2. Put the powers equal to one another.
3. Solve for the variable (usually x.)

1. $25^x = 5^{6-x} \Rightarrow (5^2)^x = 5^{6-x}$

$\Rightarrow 5^{2x} = 5^{6-x}$

2. $\therefore 2x = 6 - x$

3. $\Rightarrow 2x + x = 6 \Rightarrow 3x = 6$

$\therefore x = 2$

2001

2 (c) Solve each of the following equations for p

(i) $9^p = \frac{1}{\sqrt{3}}$

(ii) $2^{3p-7} = 2^6 - 2^5$.

SOLUTION

2 (c) (i)

$9^p = \frac{1}{\sqrt{3}}$ [Change everything to base 3.]

$\Rightarrow (3^2)^p = \frac{1}{3^{\frac{1}{2}}}$ [Use the power rules.]

$\Rightarrow 3^{2p} = 3^{-\frac{1}{2}}$ [Equate the powers.]

$\Rightarrow 2p = -\frac{1}{2}$

$\therefore p = -\frac{1}{4}$

POWER RULES

1. $a^m \times a^n = a^{m+n}$ **Ex.** $x^3 \times x^2 = x^5$

2. $\frac{a^m}{a^n} = a^{m-n}$ **Ex.** $\frac{x^5}{x^3} = x^2$

3. $a^0 = 1$ **Ex.** $5^0 = 1$

4. $a^{-n} = \frac{1}{a^n}$ **Ex.** $x^{-3} = \frac{1}{x^3}$

5. $(a^m)^n = a^{mn}$ **Ex.** $(x^3)^2 = x^6$

6. $\sqrt{a} = a^{\frac{1}{2}}$ **Ex.** $\sqrt{9} = 9^{\frac{1}{2}} = 3$

CONT...

2 (c) (ii)

$2^{3p-7} = 2^6 - 2^5$ [Work out the right hand side by calculating each number.]

$\Rightarrow 2^{3p-7} = 64 - 32 \Rightarrow 2^{3p-7} = 32$ [Change everything to base 2.]

$\Rightarrow 2^{3p-7} = 2^5$ [Equate the powers.]

$\Rightarrow 3p - 7 = 5 \Rightarrow 3p = 12$

$\therefore p = 4$

2000

2 (c) Write as a power of 3

(i) 243

(ii) $\sqrt{27}$.

Hence, solve for x the equation $\sqrt{3}(3^x) = \left(\frac{243}{\sqrt{27}}\right)^2$.

SOLUTION

2 (c) (i)

$243 = 3^5$

2 (c) (ii)

$\sqrt{27} = (27)^{\frac{1}{2}} = (3^3)^{\frac{1}{2}} = 3^{\frac{3}{2}}$

$\sqrt{3}(3^x) = \left(\frac{243}{\sqrt{27}}\right)^2 \Rightarrow 3^{\frac{1}{2}} \times 3^x = \left(\frac{3^5}{3^{\frac{3}{2}}}\right)^2$

$\Rightarrow 3^{x+\frac{1}{2}} = (3^{\frac{7}{2}})^2$

$\Rightarrow 3^{x+\frac{1}{2}} = 3^7$

$\Rightarrow x + \frac{1}{2} = 7 \Rightarrow x = 7 - \frac{1}{2}$

$\therefore x = \frac{13}{2}$

POWER RULES

1. $a^m \times a^n = a^{m+n}$	**Ex.** $x^3 \times x^2 = x^5$
2. $\frac{a^m}{a^n} = a^{m-n}$	**Ex.** $\frac{x^5}{x^3} = x^2$
3. $a^0 = 1$	**Ex.** $5^0 = 1$
4. $a^{-n} = \frac{1}{a^n}$	**Ex.** $x^{-3} = \frac{1}{x^3}$
5. $(a^m)^n = a^{mn}$	**Ex.** $(x^3)^2 = x^6$
6. $\sqrt{a} = a^{\frac{1}{2}}$	**Ex.** $\sqrt{9} = 9^{\frac{1}{2}} = 3$

1999

2 (b) Write as a power of 2

(i) 8

(ii) $8^{\frac{4}{3}}$.

Solve for x the equation

$$8^{\frac{4}{3}} = \frac{2^{5x-4}}{\sqrt{2}}.$$

SOLUTION

2 (b) (i)

$8 = 2^3$

2 (b) (ii)

$8^{\frac{4}{3}} = (2^3)^{\frac{4}{3}} = 2^4$

POWER RULES

1. $a^m \times a^n = a^{m+n}$	**Ex.** $x^3 \times x^2 = x^5$
2. $\frac{a^m}{a^n} = a^{m-n}$	**Ex.** $\frac{x^5}{x^3} = x^2$
3. $a^0 = 1$	**Ex.** $5^0 = 1$
4. $a^{-n} = \frac{1}{a^n}$	**Ex.** $x^{-3} = \frac{1}{x^3}$
5. $(a^m)^n = a^{mn}$	**Ex.** $(x^3)^2 = x^6$
6. $\sqrt{a} = a^{\frac{1}{2}}$	**Ex.** $\sqrt{9} = 9^{\frac{1}{2}} = 3$

1. $8^{\frac{4}{3}} = \frac{2^{5x-4}}{\sqrt{2}} \Rightarrow 2^4 = \frac{2^{5x-4}}{2^{\frac{1}{2}}}$

$\Rightarrow 2^4 = 2^{5x-\frac{9}{2}}$

2. $\Rightarrow 4 = 5x - \frac{9}{2}$

3. $\Rightarrow 4 + \frac{9}{2} = 5x \Rightarrow \frac{17}{2} = 5x$

$\therefore x = \frac{17}{10}$

STEPS

1. Tidy up both sides using the Power Rules until you have the same base and nothing else on both sides.
2. Put the powers equal to one another.
3. Solve for the variable (usually x.)

1998

2 (c) (i) Write $\sqrt{125}$ as a power of 5.

(ii) Solve for x the equation

$$\frac{5^{2x+1}}{\sqrt{5}} = \left(\frac{1}{\sqrt{125}}\right)^3.$$

SOLUTION

2 (c) (i)

$\sqrt{125} = 125^{\frac{1}{2}} = (5^3)^{\frac{1}{2}} = 5^{\frac{3}{2}}$

POWER RULES

1. $a^m \times a^n = a^{m+n}$	**Ex.** $x^3 \times x^2 = x^5$
2. $\frac{a^m}{a^n} = a^{m-n}$	**Ex.** $\frac{x^5}{x^3} = x^2$
3. $a^0 = 1$	**Ex.** $5^0 = 1$
4. $a^{-n} = \frac{1}{a^n}$	**Ex.** $x^{-3} = \frac{1}{x^3}$
5. $(a^m)^n = a^{mn}$	**Ex.** $(x^3)^2 = x^6$
6. $\sqrt{a} = a^{\frac{1}{2}}$	**Ex.** $\sqrt{9} = 9^{\frac{1}{2}} = 3$

2 (c) (ii)

1. $\frac{5^{2x+1}}{\sqrt{5}}=\left(\frac{1}{\sqrt{125}}\right)^3$

$\Rightarrow \frac{5^{2x+1}}{5^{\frac{1}{2}}}=\left(\frac{1}{5^{\frac{3}{2}}}\right)^3$

$\Rightarrow 5^{2x+\frac{1}{2}}=(5^{-\frac{3}{2}})^3$

$\Rightarrow 5^{2x+\frac{1}{2}}=5^{-\frac{9}{2}}$

2. $\Rightarrow 2x+\frac{1}{2}=-\frac{9}{2}$

3. $\Rightarrow 2x=-\frac{9}{2}-\frac{1}{2}$

$\Rightarrow 2x=-\frac{10}{2}=-5$

$\Rightarrow x=-\frac{5}{2}$

Steps

1. Tidy up both sides using the Power Rules until you have the same base and nothing else on both sides.
2. Put the powers equal to one another.
3. Solve for the variable (usually *x*.)

1996

2 (b) Write as a power of 2

(i) 16

(ii) $\sqrt{8}$.

Solve for x the equation

$$2^{2x-1}=\left(\frac{16}{\sqrt{8}}\right)^3.$$

Solution

2 (b) (i)

$16=2^4$

2 (b) (ii)

$\sqrt{8}=8^{\frac{1}{2}}=(2^3)^{\frac{1}{2}}=2^{\frac{3}{2}}$

1. $2^{2x-1}=\left(\frac{16}{\sqrt{8}}\right)^3 \Rightarrow 2^{2x-1}=\left(\frac{2^4}{2^{\frac{3}{2}}}\right)^3$

$\Rightarrow 2^{2x-1}=(2^{\frac{5}{2}})^3$

$\Rightarrow 2^{2x-1}=2^{\frac{15}{2}}$

2. $\Rightarrow 2x-1=\frac{15}{2}$

3. $\Rightarrow 2x=\frac{15}{2}+1 \Rightarrow 2x=\frac{17}{2}$

$\Rightarrow x=\frac{17}{4}$