ALGEBRA (Q 2 & 3, PAPER 1)

2011

2. **(a)** Given that $3a(x+5)=114$, find the value of x when $a=4$.

(b) **(i)** Find A, the solution set of $3x-5<7,\ x\in\mathbb{Z}$.

(ii) Find B, the solution set of $\frac{-2-3x}{4}\leq 1,\ x\in\mathbb{Z}$.

(iii) List the elements of $A\cap B$.

(c) Let $f(x)=x^3-2x^2+cx+d$.

(i) Given that $f(0)=6$, find the value of d.

(ii) Given that $f(3)=0$, find the value of c.

(iii) Hence, solve the equation $f(x)=0$.

SOLUTION

2 (a)

$3a(x+5)=114$

$a=4 \Rightarrow 3(4)(x+5)=114$

$12(x+5)=114$

$12x+60=114$

$12x=114-60$

$12x=54$

$x=\frac{54}{12}=\frac{9}{2}=4.5$

2 (b) (i)

SETS OF NUMBERS
Z: Set of integers. These are whole numbers that are positive and negative.
Z = {.....−3, −2, −1, 0, 1, 2, 3,.....}

$3x-5<7,\ x\in\mathbb{Z}$

$3x<7+5$

$3x<12$

$x<\frac{12}{3}$

$x<4$

$A=\{\ldots-4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3\}$

2 (b) (ii)

$\frac{-2-3x}{4} \leq 1,\ x \in \mathbb{Z}$

$-2-3x \leq 4$

$-3x \leq 4+2$

$-3x \leq 6$

$x \geq \frac{6}{-3}$

$x \geq -2$

$B = \{-2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4.......\}$

2 (b) (iii)

$A \cap B$: A intersection B (The elements common to sets A and B.)

$A = \{...-4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3\}$

$B = \{-3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4.......\}$

$A \cap B = \{-2,\ -1,\ 0,\ 1,\ 2,\ 3\}$

2 (c) (i)

$f(x) = x^3 - 2x^2 + cx + d$

$f(0) = (0)^3 - 2(0)^2 + c(0) + d$

$\quad = 0+0+0+d$

$\quad = d$

$f(0) = 6 \Rightarrow d = 6$

2 (c) (ii)

$f(x) = x^3 - 2x^2 + cx + d$

$f(3) = (3)^3 - 2(3)^2 + c(3) + 6$

$\quad = 27 - 18 + 3c + 6$

$\quad = 3c + 15$

$f(3) = 0 \Rightarrow 3c + 15 = 0$

$3c = -15$

$c = -5$

2 (c) (iii) The factor theorem states that:

> If $(x-k)$ is a factor of $f(x)$ then k is a root of $f(x)=0$,
> i.e. $f(k)=0$ and vice versa.

$f(x)=x^3-2x^2-5x+6$

$f(3)=0 \Rightarrow (x-3)$ is a factor.

$$\begin{array}{r|l} & x^2+x-2 \\ \hline x-3 & x^3-2x^2-5x+6 \\ & \mp x^3 \pm 3x^2 \\ \hline & \quad x^2-5x+6 \\ & \quad \mp x^2 \pm 3x \\ \hline & \qquad -2x+6 \\ & \qquad \pm 2x \mp 6 \\ \hline & \qquad\quad 0 \end{array}$$

$f(x)=0 \Rightarrow (x-3)(x^2+x-2)=0$

$(x-3)(x+2)(x-1)=0$

$\therefore x=-2, 1, 3$

3. **(a)** Multiply $(3x-1)(2x^2+5x-4)$ and simplify your answer.

(b) (i) Solve for x and y

$$2x=13+3y$$
$$\frac{x}{2}=\frac{2-y}{5}.$$

(ii) Hence, find the value of $4(x-y^2)$.

(c) (i) Solve for x

$$\frac{x-1}{x}+\frac{x}{x+1}=\frac{1}{2},\ x\neq 0,\ x\neq -1.$$

(ii) Verify **one** of your solutions.

SOLUTION

3 (a)

$(3x-1)(2x^2+5x-4)$

$=3x(2x^2+5x-4)-1(2x^2+5x-4)$

$=6x^3+15x^2-12x-2x^2-5x+4$

$=6x^3+13x^2-17x+4$

3 (b) (i)

Steps
1. Work on each equation so that you have the x and y terms on the left-hand side and the number on the other side.
2. Eliminate either the x's or the y's.
3. Solve for the remaining letter.
4. Substitute into either of the original equations to get the other letter.

$2x = 13 + 3y \Rightarrow 2x - 3y = 13.....(\mathbf{1})$

$\frac{x}{2} = \frac{2-y}{5}$

$5x = 2(2-y)$

$5x = 4 - 2y$

$5x + 2y = 4.....(\mathbf{2})$

$2x - 3y = 13.....(\mathbf{1})(\times 2)$

$5x + 2y = 4.......(\mathbf{2})(\times 3)$

$4x - 6y = 26$

$15x + 6y = 12$

$19x \qquad = 38 \Rightarrow x = 2$

$2(2) - 3y = 13.....(\mathbf{1})$

$4 - 3y = 13$

$-3y = 9$

$y = -3$

$x = 2,\ y = -3$

3 (b) (ii)

$4(x - y^2)$

$= 4((2) - (-3)^2)$

$= 4(2 - (9))$

$= 4(2 - 9)$

$= 4(-7)$

$= -28$

3 (c) (i)

$$\frac{x-1}{x}+\frac{x}{x+1}=\frac{1}{2}$$

$$\frac{(x-1)2\cancel{x}(x+1)}{\cancel{x}}+\frac{x\times 2x\cancel{(x+1)}}{\cancel{(x+1)}}=\frac{1\times \cancel{2}x(x+1)}{\cancel{2}}$$

$$2(x-1)(x+1)+2x^2=x(x+1)$$

$$2[x(x+1)-1(x+1)]+2x^2=x^2+x$$

$$2[x^2+\cancel{x}-\cancel{x}-1]+2x^2=x^2+x$$

$$2x^2-2+2x^2-x^2-x=0$$

$$3x^2-x-2=0$$

$$(3x+2)(x-1)=0$$

$$3x+2=0\Rightarrow 3x=-2\Rightarrow x=-\tfrac{2}{3}$$

$$x-1=0\Rightarrow x=1$$

3 (c) (ii)

$x = 1$:

$$\frac{(1)-1}{(1)}+\frac{(1)}{(1)+1}$$

$$=\frac{0}{1}+\frac{1}{2}$$

$$=0+\frac{1}{2}$$

$$=\frac{1}{2}$$