

**ALGEBRA (Q 2 & 3, PAPER 1)**

**2011**

**2. (a)** Given that  $3a(x + 5) = 114$ , find the value of  $x$  when  $a = 4$ .

**(b) (i)** Find  $A$ , the solution set of  $3x - 5 < 7$ ,  $x \in \mathbb{Z}$ .

**(ii)** Find  $B$ , the solution set of  $\frac{-2-3x}{4} \leq 1$ ,  $x \in \mathbb{Z}$ .

**(iii)** List the elements of  $A \cap B$ .

**(c)** Let  $f(x) = x^3 - 2x^2 + cx + d$ .

**(i)** Given that  $f(0) = 6$ , find the value of  $d$ .

**(ii)** Given that  $f(3) = 0$ , find the value of  $c$ .

**(iii)** Hence, solve the equation  $f(x) = 0$ .

**SOLUTION**

**2 (a)**

$$3a(x + 5) = 114$$

$$a = 4 \Rightarrow 3(4)(x + 5) = 114$$

$$12(x + 5) = 114$$

$$12x + 60 = 114$$

$$12x = 114 - 60$$

$$12x = 54$$

$$x = \frac{54}{12} = \frac{9}{2} = 4.5$$

**2 (b) (i)**

**SETS OF NUMBERS**

**Z:** Set of integers. These are whole numbers that are positive and negative.

**Z** = {.....-3, -2, -1, 0, 1, 2, 3,.....}

$$3x - 5 < 7, x \in \mathbb{Z}$$

$$3x < 7 + 5$$

$$3x < 12$$

$$x < \frac{12}{3}$$

$$x < 4$$

$$A = \{ \dots -4, -3, -2, -1, 0, 1, 2, 3 \}$$

**2 (b) (ii)**

$$\frac{-2-3x}{4} \leq 1, x \in \mathbb{Z}$$

$$-2-3x \leq 4$$

$$-3x \leq 4+2$$

$$-3x \leq 6$$

$$x \geq \frac{6}{-3}$$

$$x \geq -2$$

$$B = \{-2, -1, 0, 1, 2, 3, 4, \dots\}$$

**2 (b) (iii)**

$A \cap B$ : A intersection B (The elements common to sets A and B.)

$$A = \{\dots -4, -3, -2, -1, 0, 1, 2, 3\}$$

$$B = \{-3, -2, -1, 0, 1, 2, 3, 4, \dots\}$$

$$A \cap B = \{-2, -1, 0, 1, 2, 3\}$$

**2 (c) (i)**

$$f(x) = x^3 - 2x^2 + cx + d$$

$$f(0) = (0)^3 - 2(0)^2 + c(0) + d$$

$$= 0 + 0 + 0 + d$$

$$= d$$

$$f(0) = 6 \Rightarrow d = 6$$

**2 (c) (ii)**

$$f(x) = x^3 - 2x^2 + cx + d$$

$$f(3) = (3)^3 - 2(3)^2 + c(3) + 6$$

$$= 27 - 18 + 3c + 6$$

$$= 3c + 15$$

$$f(3) = 0 \Rightarrow 3c + 15 = 0$$

$$3c = -15$$

$$c = -5$$

**2 (c) (iii)** The factor theorem states that:

If  $(x - k)$  is a factor of  $f(x)$  then  $k$  is a root of  $f(x) = 0$ ,  
i.e.  $f(k) = 0$  and vice versa.

$$f(x) = x^3 - 2x^2 - 5x + 6$$

$$f(3) = 0 \Rightarrow (x - 3) \text{ is a factor.}$$

$$\begin{array}{r} x-3 \quad \overline{) \quad x^3 - 2x^2 - 5x + 6} \\ \underline{\mp x^3 \pm 3x^2} \phantom{+ 6} \\ \phantom{x-3} \quad x^2 - 5x + 6 \\ \phantom{x-3} \quad \underline{\mp x^2 \pm 3x} \\ \phantom{x-3} \phantom{x^2} - 2x + 6 \\ \phantom{x-3} \phantom{x^2} \underline{\pm 2x \mp 6} \\ \phantom{x-3} \phantom{x^2} \phantom{- 2x} 0 \end{array}$$

$$\begin{aligned} f(x) = 0 &\Rightarrow (x-3)(x^2 + x - 2) = 0 \\ &(x-3)(x+2)(x-1) = 0 \\ \therefore x &= -2, 1, 3 \end{aligned}$$

**3. (a)** Multiply  $(3x-1)(2x^2 + 5x - 4)$  and simplify your answer.

**(b) (i)** Solve for  $x$  and  $y$

$$2x = 13 + 3y$$

$$\frac{x}{2} = \frac{2-y}{5}$$

**(ii)** Hence, find the value of  $4(x - y^2)$ .

**(c) (i)** Solve for  $x$

$$\frac{x-1}{x} + \frac{x}{x+1} = \frac{1}{2}, \quad x \neq 0, \quad x \neq -1.$$

**(ii)** Verify **one** of your solutions.

**SOLUTION**

**3 (a)**

$$\begin{aligned} &(3x-1)(2x^2 + 5x - 4) \\ &= 3x(2x^2 + 5x - 4) - 1(2x^2 + 5x - 4) \\ &= 6x^3 + 15x^2 - 12x - 2x^2 - 5x + 4 \\ &= 6x^3 + 13x^2 - 17x + 4 \end{aligned}$$

**3 (b) (i)**

**STEPS**

1. Work on each equation so that you have the  $x$  and  $y$  terms on the left-hand side and the number on the other side.
2. Eliminate either the  $x$ 's or the  $y$ 's.
3. Solve for the remaining letter.
4. Substitute into either of the original equations to get the other letter.

$$2x = 13 + 3y \Rightarrow 2x - 3y = 13 \dots (1)$$

$$\frac{x}{2} = \frac{2-y}{5}$$

$$5x = 2(2-y)$$

$$5x = 4 - 2y$$

$$5x + 2y = 4 \dots (2)$$

$$2x - 3y = 13 \dots (1)(\times 2)$$

$$5x + 2y = 4 \dots (2)(\times 3)$$

$$4x - 6y = 26$$

$$15x + 6y = 12$$

$$\hline 19x = 38 \Rightarrow x = 2$$

$$2(2) - 3y = 13 \dots (1)$$

$$4 - 3y = 13$$

$$-3y = 9$$

$$y = -3$$

$$x = 2, y = -3$$

**3 (b) (ii)**

$$\begin{aligned} & 4(x - y^2) \\ &= 4((2) - (-3)^2) \\ &= 4(2 - (9)) \\ &= 4(2 - 9) \\ &= 4(-7) \\ &= -28 \end{aligned}$$

**3 (c) (i)**

$$\frac{x-1}{x} + \frac{x}{x+1} = \frac{1}{2}$$

$$\frac{(x-1)\cancel{2}(x+1)}{\cancel{2}} + \frac{x \times \cancel{2}x(x+1)}{(x+1)} = \frac{1 \times \cancel{2}x(x+1)}{\cancel{2}}$$

$$2(x-1)(x+1) + 2x^2 = x(x+1)$$

$$2[x(x+1) - 1(x+1)] + 2x^2 = x^2 + x$$

$$2[x^2 + \cancel{x} - \cancel{x} - 1] + 2x^2 = x^2 + x$$

$$2x^2 - 2 + 2x^2 - x^2 - x = 0$$

$$3x^2 - x - 2 = 0$$

$$(3x+2)(x-1) = 0$$

$$3x+2=0 \Rightarrow 3x=-2 \Rightarrow x=-\frac{2}{3}$$

$$x-1=0 \Rightarrow x=1$$

**3 (c) (ii)**

$x = 1:$

$$\frac{(1)-1}{(1)} + \frac{(1)}{(1)+1}$$

$$= \frac{0}{1} + \frac{1}{2}$$

$$= 0 + \frac{1}{2}$$

$$= \frac{1}{2}$$