ALGEBRA (Q 2 & 3, PAPER 1)

2010

2. **(a)** Find the values of x which satisfy

$$2(3+4x) \le 22, \text{ where } x \in \mathbf{N}.$$

(b) Solve for x and y

$$2x - y = 1$$
$$x^2 - xy = -6.$$

(c) **(i)** Show, by division, that $3x + 1$ is a factor of $3x^3 + 4x^2 - 89x - 30$.

(ii) Hence, or otherwise, solve the equation $3x^3 + 4x^2 - 89x - 30 = 0$.

SOLUTION

2 (a)

$2(3+4x) \le 22$

$6 + 8x \le 22$

$8x \le 22 - 6$

$8x \le 16$

$x \le 2$

x is a natural number less than or equal to 2.
$x = \{1, 2\}$

2 (b)

SOLVING SIMULTANEOUS LINEAR EQUATIONS
1. Get a letter on its own from the linear equation.
2. Substitute into the quadratic and solve.
3. Substitute these values back into linear.

STEP 1

$2x - y = 1 \Rightarrow y = 2x - 1$.....**(1)**

STEP 2
Substitute Equation (**1**) into the quadratic.

$x^2 - x(y) = -6$

$x^2 - x(2x-1) = -6$

$x^2 - 2x^2 + x = -6$

$-x^2 + x + 6 = 0$

$x^2 - x - 6 = 0$

$(x+2)(x-3) = 0$

$\therefore x = -2, 3$

STEP 3

$y = 2(x) - 1$

$x = -2: y = 2(-2) - 1 = -4 - 1 = -5$

$x = 3: \quad y = 2(3) - 1 = 6 - 1 = 5$

ANSWER: $x = 3, -2$; $y = 5, -5$

2 (c) (i)

$$\begin{array}{r|l} & x^2+x-30 \\ \hline 3x+1 & 3x^3+4x^2-89x-30 \\ & \underline{-3x^3 \mp x^2} \\ & \quad 3x^2-89x-30 \\ & \quad \underline{-3x^2 \mp x} \\ & \qquad -90x-30 \\ & \qquad \underline{\pm 90x \pm 30} \\ & \qquad\qquad 0 \end{array}$$

As the remainder is 0, $3x + 1$ is a factor of the cubic.

2 (c) (ii)

$3x^3+4x^2-89x-30=(3x+1)(x^2+x-30)=0$

$3x^3+4x^2-89x-30=(3x+1)(x+6)(x-5)=0$

$\therefore x=-6, -\frac{1}{3}, 5$

3. **(a)** Given that $3(b + a) = t(6 - a)$, calculate the value of a when $t = 3$ and $b = -4$.

(b) Solve for x

$$5(x+1)^2=2(x+1)+5.$$

Give your answer correct to two decimal places.

(c) **(i)** $2+\sqrt{3}$ is a root of the equation $x^2-4x+c=0$, where c is a real number. Find the value of c and write down the other root.

(ii) The equation $x^2+10x+k=0$ has equal roots. Find the value of the real number k and write down the value of each root.

SOLUTION

3 (a)

$t=3,\ b=-4$

$3((b)+a)=(t)(6-a)$

$3((-4)+a)=(3)(6-a)$

$3(a-4)=3(6-a)$

$3a-12=18-3a$

$3a+3a=18+12$

$6a=30$

$\therefore a=5$

3 (b)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 5$
$b = -2$
$c = -5$

$5(x+1)^2 = 2(x+1)+5$

$5(x+1)^2 - 2(x+1) - 5 = 0$

$y \to (x+1)$ [Replace $(x + 1)$ by y and solve the quadratic for y.]

$5y^2 - 2y - 5 = 0$

$$y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(-5)}}{2(5)}$$

$$= \frac{2 \pm \sqrt{4+100}}{10}$$

$$= \frac{2 \pm \sqrt{104}}{10}$$

$= -0.82, 1.22$

$y = x+1 \Rightarrow x = y-1$ [Undo the substitution and solve for x.]

$x = -0.82 - 1 = -1.82$

$x = 1.22 - 1 = 0.22$

$x = -1.82, 0.22$

3 (c) (i)

$x^2 - 4x + c = 0$

$(2+\sqrt{3})^2 - 4(2+\sqrt{3}) + c = 0$ [Replace x by the root and solve for c.]

$(2+\sqrt{3})(2+\sqrt{3}) - 4(2+\sqrt{3}) + c = 0$

$4 + 2\sqrt{3} + 2\sqrt{3} + 3 - 8 - 4\sqrt{3} + c = 0$

$-1 + c = 0$

$\therefore c = 1$

Solve the quadratic equation to find the other root.

$a = 1$
$b = -4$
$c = 1$

$x^2 - 4x + 1 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16-4}}{2}$$

$$= \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm \sqrt{4 \times 3}}{2}$$

$$= \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

Other root: $2 - \sqrt{3}$

3 (c) (ii)

The quadratic equation $ax^2+bx+c=0$ has equal roots or one root if $b^2=4ac$.

$a=1$
$b=10$
$c=k$

$b^2=4ac \Rightarrow (10)^2=4(1)(k)$

$100=4k$

$\therefore k=25$

$x^2+10x+25=0$

$(x+5)(x+5)=0$

$\therefore x=-5$