ALGEBRA (Q 2 & 3, PAPER 1)

2009

2 (a) Find the value of $\frac{3x-2y-1}{5}$ when $x = 13$ and $y = 14$.

(b) (i) Find the value of 3^6.

(ii) Write 27 in the form 3^k, where $k \in \mathbf{N}$.

(iii) Find the value of x for which $27 \times 3^x = \frac{1}{729}$.

(c) Let $f(x) = x^3 + x^2 - 4x - 4$.

(i) Verify that $f(-2) = 0$.

(ii) Solve the equation

$$x^3 + x^2 - 4x - 4 = 0.$$

SOLUTION

2 (a)

$$\frac{3x-2y-1}{5} = \frac{3(13)-2(14)-1}{5} = \frac{39-28-1}{5} = \frac{10}{5} = 2$$

2 (b) (i)

$$3^6 = 3\times3\times3\times3\times3\times3 = 729$$

2 (b) (ii)

$$27 = 3\times3\times3 = 3^3$$

2 (b) (iii)

$$27\times3^x = \frac{1}{729}$$

$$3^3\times3^x = \frac{1}{3^6}$$

$$3^{x+3} = 3^{-6}$$

$$\Rightarrow x+3 = -6$$

$$\therefore x = -9$$

POWER RULES

1. $a^m \times a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $a^0 = 1$
4. $a^{-n} = \frac{1}{a^n}$
5. $(a^m)^n = a^{mn}$
6. $\sqrt{a} = a^{\frac{1}{2}}$

2 (c) (i)

$f(x) = x^3 + x^2 - 4x - 4$

$f(-2) = (-2)^3 + (-2)^2 - 4(-2) - 4$

$f(-2) = -8 + 4 + 8 - 4 = 0$

2 (c) (ii)

STEPS TO SOLVING A CUBIC

1. Find a root by guessing.
2. Get a factor from the root.
3. Divide the factor into the cubic.
4. Factorise the resulting quadratic.
5. Write down the three roots.

STEP 1 already done.
STEP 2: (x + 2) is a factor.
STEP 3:

$$\begin{array}{r|l} & x^2 - x - 2 \\ \hline x+2 & x^3 + x^2 - 4x - 4 \\ & \underline{-x^3 \mp 2x^2} \\ & \quad -x^2 - 4x - 4 \\ & \quad \underline{\pm x^2 \pm 2x} \\ & \qquad -2x - 4 \\ & \qquad \underline{\pm 2x \pm 4} \\ & \qquad\quad 0 \end{array}$$

STEP 4: $x^2 - x - 2 = (x+1)(x-2)$

STEP 5: $x^3 + x^2 - 4x - 4 = (x^2 - x - 2)(x+2) = (x+1)(x-2)(x+2) = 0$

$\therefore x = -2, -1, 2$

3 (a) Simplify $x(2x+7)-3(x-4)$.

(b) (i) Solve for x and y

$$x+y=7$$
$$x^2+y^2=29.$$

(ii) Which one of the values of y in **(i)** above satisfies the inequality

$$6-2y<0?$$

Justify your answer.

(c) A rectangle has length $2\sqrt{x}$ cm and width $\sqrt{x}$ cm.

The length of a diagonal of the rectangle is $\sqrt{45}$ cm.

(i) Find the area of the rectangle.

(ii) The area of a square is twice the area of the rectangle.
Find the length of a side of the square.

SOLUTION

3 (a)

$x(2x+7)-3(x-4)$

$=2x^2+7x-3x+12$

$=2x^2+4x+12$

3 (b) (i)

SOLVING SIMULTANEOUS LINEAR & QUADRATIC EQUATIONS
1. Get a letter on its own from the linear equation.
2. Substitute into the quadratic and solve.
3. Substitute these values back into linear.

$x+y=7 ...$ **(L)**

$x^2+y^2=29 ...$ **(Q)**

1. $x+y=7 \Rightarrow x=(7-y)$

2. $x^2+y^2=29 \Rightarrow (7-y)^2+y^2=29$

$(7-y)(7-y)+y^2=29$

$49-7y-7y+y^2+y^2-29=0$

$2y^2-14y+20=0$

$y^2-7y+10=0$

$(y-2)(y-5)=0$

$\therefore y=2, 5$

3. $y=2: \; x=7-y=7-2=5$

$y=5: \; x=7-y=7-5=2$

ANSWER: (2, 5), (5, 2)

3 (b) (ii)

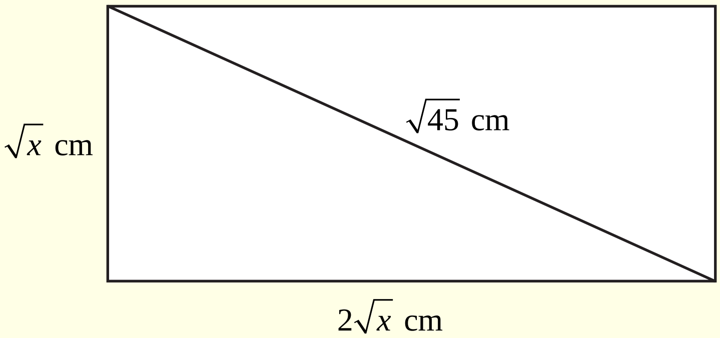
$6-2y<0$

$6-2(2)=6-4=2<0$ (False)

$6-2(5)=6-10=-4<0$ (True)

Answer: $y = 5$

3 (c) (i)



The triangle is right-angled so apply Pythagoras.

$x^2 + y^2 = r^2$

$(\sqrt{x})^2 + (2\sqrt{x})^2 = (\sqrt{45})^2$

$x + 4x = 45$

$5x = 45$

$x = 9$

$\text{Length} = 2\sqrt{x} = 2\sqrt{9} = 6 \text{ cm}$

$\text{Width} = \sqrt{x} = \sqrt{9} = 3 \text{ cm}$

$\text{Area} = \text{Length} \times \text{Width} = 6 \times 3 = 18 \text{ cm}^2$

3 (c) (ii)

Area of square = 36 cm^2

$\text{Area} = l \times l = l^2 = 36 \text{ cm}^2$

$\therefore l = \sqrt{36} = 6 \text{ cm}$