ALGEBRA (Q 2 & 3, PAPER 1)

2008

2 (a) Simplify $3(4x+5)-2(6x+4)$.

(b) (i) Solve $x^2-4x+1=0$.

Write your solutions in the form $a\pm\sqrt{b}$, where $a, b\in\mathbf{N}$.

(ii) Find the value of x for which

$$\frac{5^x}{3}=\frac{5^6}{75}.$$

(c) (i) Factorise x^2+4x+4.

(ii) Simplify $\sqrt{x^2+4x+4}+\sqrt{x^2+2x+1}$, given that $x\geq 0$.

(iii) Given that $x\geq 0$, solve for x

$$\sqrt{x^2+4x+4}+\sqrt{x^2+2x+1}=x^2.$$

SOLUTION

2 (a)

$3(4x+5)-2(6x+4)$

$=12x+15-12x-8$

$=7$

Multiply every term by every term and then tidy up by adding and subtracting like terms.

2 (b) (i)

$a=1$

$b=-4$

$c=1$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}=\frac{4\pm\sqrt{16-4}}{2}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x=\frac{4\pm\sqrt{12}}{2}=\frac{4\pm\sqrt{4\times 3}}{2}$$

$$\Rightarrow x=\frac{4\pm 2\sqrt{3}}{2}$$

$$\therefore x=2\pm\sqrt{3}$$

2 (b) (ii) Solving Power Equations

Steps
1. Tidy up both sides using the Power Rules until you have the same base and nothing else on both sides.
2. Put the powers equal to one another.
3. Solve for the variable (usually x.)

Power Rules

1. $a^m \times a^n = a^{m+n}$
2. $\dfrac{a^m}{a^n} = a^{m-n}$
3. $a^0 = 1$
4. $a^{-n} = \dfrac{1}{a^n}$
5. $(a^m)^n = a^{mn}$
6. $\sqrt{a} = a^{\frac{1}{2}}$

$\dfrac{5^x}{3} = \dfrac{5^6}{75}$ [Multiply across by 75.]

$\Rightarrow 25 \times 5^x = 5^6$ [Change 25 to a power of 5.]

$\Rightarrow 5^2 \times 5^x = 5^6$ [Use Power Rule No. 1]

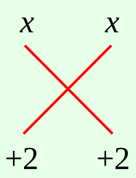
$\Rightarrow 5^{x+2} = 5^6$ [Equate the powers.]

$\therefore x + 2 = 6 \Rightarrow x = 6 - 2$

$\therefore x = 4$

2 (c) (i)

$x^2 + 4x + 4 = (x+2)(x+2) = (x+2)^2$



2 (c) (ii)

$x^2 + 2x + 1 = (x+1)(x+1) = (x+1)^2$

$\therefore \sqrt{x^2 + 4x + 4} + \sqrt{x^2 + 2x + 1}$

$= \sqrt{(x+2)^2} + \sqrt{(x+1)^2}$ [Taking the square root of a square leaves you with the bracket.]

$= (x+2) + (x+1)$

$= 2x + 3$

2 (c) (iii)

$\therefore \sqrt{x^2 + 4x + 4} + \sqrt{x^2 + 2x + 1} = x^2$

$\Rightarrow 2x + 3 = x^2$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow (x-3)(x+1) = 0$

$\therefore x = 3, \cancel{-1}$

Take the positive solution as you are told that x is greater than or equal to zero.

3 (a) Given that $a(x+5)=8$, express x in terms of a.

(b) (i) Solve for x and y

$$x-y=1$$
$$x^2+y^2=25.$$

(ii) Hence, find the two possible values of $x-y^2$.

(c) (i) Let $f(x)=x^2+bx+c,\ x\in\mathbf{R}$.
The graph of the function f intersects the y-axis at 3 and the x-axis at –1.
Find the value of b and the value of c.

(ii) The lengths of the sides of an isosceles triangle are $\sqrt{x^2+1}$, $\sqrt{x^2+1}$ and $2x$.
Taking $2x$ as the base, find the perpendicular height of the triangle.

SOLUTION

3 (a)

$a(x+5)=8$ [Multiply out the bracket.]

$\Rightarrow ax+5a=8$ [Isolate the ax term.]

$\Rightarrow ax=8-5a$ [Divide both sides by a to isolate x.]

$$\Rightarrow x=\frac{8-5a}{a}$$

3 (b) (i)

SOLVING SIMULTANEOUS LINEAR & QUADRATIC EQUATIONS
1. Get a letter on its own from the linear equation.
2. Substitute into the quadratic and solve.
3. Substitute these values back into linear.

1. $x-y=1\Rightarrow x=y+1$

2. $x^2+y^2=25\Rightarrow (y+1)^2+y^2=25$

$\Rightarrow y^2+2y+1+y^2=25$

$\Rightarrow 2y^2+2y-24=0$

$\Rightarrow y^2+y-12=0$

$\Rightarrow (y+4)(y-3)=0$

$\therefore y=-4,\ 3$

3. $y=-4: x=(-4)+1=-3\Rightarrow(-3,\ -4)$

$y=3: x=(3)+1=4\Rightarrow(4,\ 3)$

3 (b) (ii)

$x=-3,\ y=-4\Rightarrow x-y^2=(-3)-(-4)^2=-3-16=-19$

$x=4,\ y=3\Rightarrow x-y^2=(4)-(3)^2=4-9=-5$

3 (c) (i)

Intersects y-axis: (0, 3) is on the curve $\Rightarrow 3=(0)^2+b(0)+c \Rightarrow c=3$

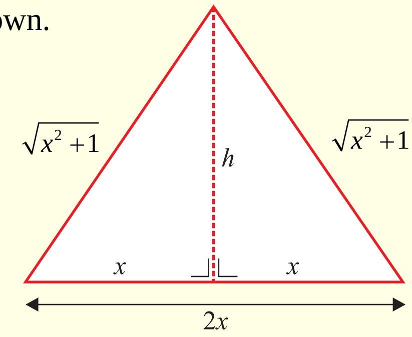
Intersects x-axis: (–1, 0) is on the curve $\Rightarrow 0=(-1)^2+b(-1)+(3)$

$\Rightarrow 0=1-b+3$

$\therefore b=4$

3 (c) (ii)

Use Pythagoras on one of the right-angled triangles shown.



$x^2+y^2=r^2$

$\therefore x^2+h^2=(\sqrt{x^2+1})^2$

$\Rightarrow x^2+h^2=x^2+1$

$\Rightarrow h^2=x^2-x^2+1$

$\Rightarrow h^2=1$

$\therefore h=1$