ALGEBRA (Q 2 & 3, PAPER 1)

2007

2 (a) Find the solution set of $4x-15<1,\ x\in\mathbf{N}$.

(b) (i) Find the value of $\dfrac{x+3y+5}{2x+2y}$ when $x=\frac{5}{2}$ and $y=\frac{1}{3}$.

(ii) Find the value of x for which $2^{x+3}=4^x$.

(c) (i) Solve the equation $x-\frac{1}{x}=2$ and write your solutions in the form $a\pm\sqrt{b}$, where $a,b\in\mathbf{N}$.

(ii) Verify **one** of your solutions.

SOLUTION

2 (a)

$4x-15<1 \Rightarrow 4x<1+15$

$\Rightarrow 4x<16 \Rightarrow x<4$

Solution set x = {0, 1, 2, 3}

N: Set of natural numbers. These are whole positive numbers. **N** = {0, 1, 2, 3,...}

2 (b) (i)

$$\frac{x+3y+5}{2x+2y}=\frac{(\frac{5}{2})+3(\frac{1}{3})+5}{2(\frac{5}{2})+2(\frac{1}{3})}=\frac{\frac{5}{2}+1+5}{5+\frac{2}{3}}$$

$$=\frac{\frac{17}{2}}{\frac{17}{3}}=\frac{17}{2}\times\frac{3}{17}=\frac{3}{2}$$

2 (b) (ii)

STEPS
1. Tidy up both sides using the Power Rules until you have the same base and nothing else on both sides.
2. Put the powers equal to one another.
3. Solve for the variable (usually x.)

1. $2^{x+3}=4^x \Rightarrow 2^{x+3}=(2^2)^x \Rightarrow 2^{x+3}=2^{2x}$

2. $x+3=2x$

3. $3=2x-x \Rightarrow x=3$

POWER RULES

5. $(a^m)^n=a^{mn}$ **Ex.** $(x^3)^2=x^6$

2 (c) (i)

$x - \frac{1}{x} = 2$ [Multiply across by x.]

$x^2 - 1 = 2x \Rightarrow x^2 - 2x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ 2

$a = 1$
$b = -2$
$c = -1$

$$\therefore x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm \sqrt{8}}{2}$$

$$= \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

Note: $\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$

$\therefore x = 1 + \sqrt{2}$ and $x = 1 - \sqrt{2}$.

2 (c) (ii)

Verify $x = 1 - \sqrt{2}$ is a solution by sustituting it back into the original equation.

$x - \frac{1}{x} = 2 \Rightarrow (1+\sqrt{2}) - \frac{1}{(1+\sqrt{2})} = 2$ [This needs to be proved]

$$(1+\sqrt{2}) - \frac{1}{(1+\sqrt{2})} \times \frac{(1-\sqrt{2})}{(1-\sqrt{2})}$$ [Multiply above and below by the conjugate of the denominator.]

$$= (1+\sqrt{2}) - \frac{1-\sqrt{2}}{1-\sqrt{2}+\sqrt{2}-2} = (1+\sqrt{2}) - \frac{1-\sqrt{2}}{-1}$$

$$= 1 + \sqrt{2} + 1 - \sqrt{2} = 2$$

3 (a) Solve $2x = 3(5 - x)$.

(b) Solve the simultaneous equations

$\frac{x}{4} - \frac{y}{3} = \frac{5}{6}$

$2x - 6 = 3y$.

(c) Let $f(x) = 2x^3 + 11x^2 + 4x - 5$.

(i) Verify that $f(-1) = 0$.

(ii) Solve the equation

$2x^3 + 11x^2 + 4x - 5 = 0$.

SOLUTION

3 (a)

$2x = 3(5 - x) \Rightarrow 2x = 15 - 3x$

$\Rightarrow 2x + 3x = 15$

$\Rightarrow 5x = 15$

$\Rightarrow x = \frac{15}{5}$

$\therefore x = 3$

3 (b)

Steps
1. Work on each equation so that you have the *x* and *y* terms on the left-hand side and the number on the other side.
2. Eliminate either the *x*'s or the *y*'s.
3. Solve for the remaining letter.
4. Substitute into either of the original equations to get the other letter.

1. $\frac{x}{4}-\frac{y}{3}=\frac{5}{6}(\times 12) \Rightarrow 3x-4y=10....(\mathbf{1})$

$2x-6=3y \Rightarrow 2x-3y=6...(\mathbf{2})$

2. Eliminate the *x*'s:

$3x-4y=10....(\mathbf{1})(\times 2)$
$2x-3y=6......(\mathbf{2})(\times -3)$

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$$\begin{aligned} 6x-8y&=20 \\ -6x+9y&=-18 \\ \hline y&=2 \end{aligned}$$

3. $y=2$

4. Substitute into equation (1): $2x-3y=6 \Rightarrow 2x-3(2)=6$

$\Rightarrow 2x-6=6 \Rightarrow 2x=12$

$\therefore x=6$

3 (c) (i)

$f(x)=2x^3+11x^2+4x-5$

$\therefore f(-1)=2(-1)^3+11(-1)^2+4(-1)-5=2(-1)+11(1)-4-5$

$=-2+11-4-5=0$

3 (c) (ii)

Steps
1. Guess at a root (unless a root is given) by substituting in numbers $0, 1, -1, 2, -2, ...$ until you get zero.
2. Using the factor theorem, form a factor from the root.
3. Divide the cubic by the factor to get the quadratic.
4. Solve the quadratic by factorising or using formula **2**.
5. Write down the three roots.

1. $f(-1)=0 \Rightarrow -1$ is a root.

2. $\therefore (x+1)$ is a factor.

3. Divide the factor into the cubic as shown to the right.

$\therefore 2x^3+11x^2+4x-5=(x+1)(2x^2+9x-5)=0$

$$\begin{array}{r|l} & 2x^2+9x-5 \\ \hline x+1 & 2x^3+11x^2+4x-5 \\ & \mp 2x^3 \mp 2x^2 \\ \hline & \quad 9x^2+4x-5 \\ & \quad \mp 9x^2 \mp 9x \\ \hline & \qquad -5x-5 \\ & \qquad \pm 5x \pm 5 \\ \hline & \qquad\qquad 0 \end{array}$$

4. The resulting quadratic can be factorised.

$2x^2+9x-5=(2x-1)(x+5)$

5. $2x^3+11x^2+4x-5=(x+1)(2x-1)(x+5)=0$

Set each factor equal to zero and solve for *x*.

$\therefore x=-5, -1, \frac{1}{2}$