ALGEBRA (Q 2 & 3, PAPER 1)

2006

2 (a) Simplify $3(2x+4)-5(x+1)$.

(b) Let $f(x)=2x^3+ax^2+bx+14$.

(i) Express $f(2)$ in terms of a and b.

(ii) If $f(2)=0$ and $f(-1)=0$, find the value of a and the value of b.

(c) (i) Find the smallest natural number k such that
$2x+4(x+3)+7(2x+4)<20(x+k)$.

(ii) The lengths of the sides of a triangle are
$4\sqrt{x}$, $(x-4)$ and $(x+4)$, where $x>4$.
Prove that the triangle is right-angled.

SOLUTION

2 (a)

$3(2x+4)-5(x+1)$

$=6x+12-5x-5$

$=x+7$

> Multiply every term by every term and then tidy up by adding and subtracting like terms.

2 (b) (i)

$f(x)=2x^3+ax^2+bx+14$

$\Rightarrow f(2)=2(2)^3+a(2)^2+b(2)+14=16+4a+2b+14$

$\therefore f(2)=4a+2b+30$

2 (b) (ii)

$f(2)=0\Rightarrow 4a+2b+30=0\Rightarrow 2a+b=-15....(\mathbf{1})$

$f(-1)=0\Rightarrow 2(-1)^3+a(-1)^2+b(-1)+14=0$

$\Rightarrow -2+a-b+14=0\Rightarrow a-b=-12...(\mathbf{2})$

Solve equations (**1**) and (**2**) simultaneously.

$2a+b=-15...(\mathbf{1})$

$a-b=-12....(\mathbf{2})$

$3a \quad =-27\Rightarrow a=-9$

Substitute this value of a into equation (**2**): $(-9)-b=-12\Rightarrow b=3$

2 (c) (i)

$2x+4(x+3)+7(2x+4)<20(x+k)$ [Multiply out the brackets.]

$\Rightarrow 2x+4x+12+14x+28<20x+20k$ [Bring the x's to the right and the numbers to the left.]

$\Rightarrow 12+28-20k<20x-2x-4x-14x$

$\Rightarrow 40-20k<0$

$\Rightarrow 40<20k \Rightarrow 2<k \Rightarrow k>2$

The smallest natural number (whole, positive number) greater than 2 is 3.

$\therefore k=3$

2 (c) (ii)

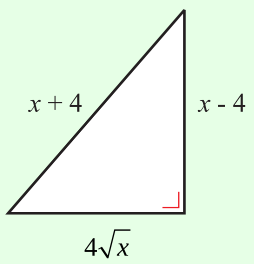
The longest side is the hypotenuse. How do you know which side is the longest?

Put a number like 9 in for x and you will see which is the longest side.

If it is right-angled, you need to show that

$(4\sqrt{x})^2+(x-4)^2=(x+4)^2$.

$x^2+y^2=r^2$ 1



LHS	*RHS*
$(4\sqrt{x})^2+(x-4)^2$	$(x+4)^2$
$=16x+x^2-8x+16$	$=x^2+8x+16$
$=x^2+8x+16$	

3 (a) Find the value of $\dfrac{ab-c}{2}$ when $a=3$, $b=\frac{2}{3}$ and $c=1$.

(b) Solve for x and y

$x-2y=10$

$x^2+y^2=20.$

(c) Solve for x

$x=\dfrac{3+2x}{x-2},\ x\neq 2$

and give your solutions in the form form $a\pm\sqrt{b}$, where $a,b\in\mathbf{N}$.

Write one of your solutions correct to two decimal places. Using this value, show that the difference between the values of the left hand side and the right hand side of the given equation is less that 0.1.

Solution

3 (a)

$$\frac{ab-c}{2}=\frac{(3)(\frac{2}{3})-1}{2}=\frac{2-1}{2}=\frac{1}{2}$$

3 (b)

STEPS
1. Eliminate a letter from the linear equation.
2. Substitute into quadratic and solve for the other letter.
3. Substitute these values into the linear to get all solutions.

1. Isolate the x as this is the easier letter to get on its own.

$x-2y=10$
$\Rightarrow x=2y+10\ldots(\mathbf{A})$

2.

$x^2+y^2=20$
$\Rightarrow (2y+10)^2+y^2=20$
$\Rightarrow 4y^2+40y+100+y^2=20$
$\Rightarrow 5y^2+40y+80=0$
$\Rightarrow y^2+8y+16=0$
$\Rightarrow (y+4)(y+4)=0$
$\therefore y=-4$

3. Substitute this value of y into Eqn. (**A**) to obtain the x value.

$y=-4: x=2(-4)+10=2$

ANSWER: $(2, -4)$

3 (c)

$x=\dfrac{3+2x}{x-2}\Rightarrow x(x-2)=3+2x$ [Multiply both sides by $(x-2)$.]

$\Rightarrow x^2-2x=3+2x$

$\Rightarrow x^2-4x-3=0$ [Solve using formula **2**.]

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad \ldots\ldots\ \mathbf{2}$$

$a=1$
$b=-4$
$c=-3$

$$\therefore x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-3)}}{2(1)}=\frac{4\pm\sqrt{16+12}}{2}=\frac{4\pm\sqrt{28}}{2}$$

$$=\frac{4\pm2\sqrt{7}}{2}=2\pm\sqrt{7}$$

Note: $\sqrt{28}=\sqrt{4\times7}=2\sqrt{7}$

Write $2+\sqrt{7}$ in decimal form: $2+\sqrt{7}=4.65$

LHS: $x = 4.65$

RHS: $\dfrac{3+2(4.65)}{4.65-2}=4.64$ [Using your calculator.]

Subtracting both sides: $4.65-4.64=0.01<0.1$