ALGEBRA (Q 2 & 3, PAPER 1)

2005

2 (a) Find the value of $x^2 - 5xy$ when $x = 3$ and $y = -2$.

(b) Solve for x and y

$$x + 3 = 2y$$
$$xy - 7y + 8 = 0.$$

(c) (i) Write $\sqrt{x} + \frac{1}{\sqrt{x}}$ as a single fraction.

(ii) Hence, or otherwise, simplify $\left(\frac{2\sqrt{x}}{1+x}\right)\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$.

(iii) Solve for x

$$\left(\frac{2\sqrt{x}}{1+x}\right)\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) = x - 3.$$

SOLUTION

2 (a)

$$x^2 - 5xy = (3)^2 - 5(3)(-2) = 9 + 30 = 39$$

2 (b)

STEPS
1. Eliminate a letter from the linear equation.
2. Substitute into quadratic and solve for the other letter.
3. Substitute these values into the linear to get all solutions.

1. Isolate the x as this is the easier letter to get on its own.

$$x + 3 = 2y$$
$$\Rightarrow x = 2y - 3 \ldots \textbf{(A)}$$

2.

$$xy - 7y + 8 = 0$$
$$\Rightarrow (2y - 3)y - 7y + 8 = 0$$
$$\Rightarrow 2y^2 - 3y - 7y + 8 = 0$$
$$\Rightarrow 2y^2 - 10y + 8 = 0$$
$$\Rightarrow y^2 - 5y + 4 = 0$$
$$\Rightarrow (y - 1)(y - 4) = 0$$
$$\Rightarrow y = 1,\ y = 4$$

3. Substitute the values of y into Eqn. (**A**) to obtain the x values.

$y = 1$: $x = 2(1) - 3 = 2 - 3 = -1$

$y = 4$: $x = 2(4) - 3 = 8 - 3 = 5$

ANSWER: $(-1, 1)$, $(5, 4)$

2 (c) (i)

$$\frac{\sqrt{x}}{1}+\frac{1}{\sqrt{x}}=\frac{\sqrt{x}\sqrt{x}+1}{\sqrt{x}}=\frac{x+1}{\sqrt{x}}$$

2 (c) (ii)

$$\left(\frac{2\sqrt{x}}{1+x}\right)\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)=\left(\frac{2\sqrt{x}}{x+1}\right)\left(\frac{x+1}{\sqrt{x}}\right)=2$$

2 (c) (iii)

$$\left(\frac{2\sqrt{x}}{1+x}\right)\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)=x-3\Rightarrow 2=x-3$$

$\therefore x=5$

3 (a) Given that $ax + b = c$, express x in terms of a, b and c, where $a \neq 0$.

(b) (i) Find A, the solution set of $3x-2\leq 4,\ x\in \mathbf{Z}$.

(ii) Find B, the solution set of $\frac{1-3x}{2}<5,\ x\in \mathbf{Z}$.

(iii) List the elements of $A\cap B$.

(c) Let $f(x)=2x^3-3x^2-11x+6$.

(i) Verify that $f(3) = 0$.

(ii) Solve the equation

$$2x^3-3x^2-11x+6=0.$$

SOLUTION

3 (a)

$ax+b=c\Rightarrow ax=c-b$ [Isolate the ax term by moving the b term across.]

$\Rightarrow x=\frac{c-b}{a}$ [Isolate the x term by dividing both sides by a.]

3 (b) (i)

$3x-2\leq 4\Rightarrow 3x\leq 6\Rightarrow x\leq 2$

$\therefore A=\{.....-3,\ -2,\ -1,\ 0,\ 1,\ 2\}$

Z: Set of integers. These are whole numbers that are positive and negative.

Z = {.....−3, −2, −1, 0, 1, 2, 3,.....}

3 (b) (ii)

$\frac{1-3x}{2} < 5 \Rightarrow 1-3x = 10$ [Multiply both sides by 5.]

$\Rightarrow -3x < 10-1$ [Move the numbers to the right.]

$\Rightarrow -3x < 9$ [Divide across by -2. Remember to reverse the inequality.]

$\Rightarrow x > -3$

$\therefore B = \{-2, -1, 0, 1, 2, 3, ...\}$

3 (b) (iii)

$A = \{.....-3, -2, -1, 0, 1, 2\}$

$B = \{-2, -1, 0, 1, 2, 3, ...\}$

$\therefore A \cap B = \{-2, -1, 0, 1, 2\}$

> $A \cap B$: A intersection B (The elements common to sets A and B.)

3 (c) (i)

$f(x) = 2x^3 - 3x^2 - 11x + 6$

$\therefore f(3) = 2(3)^3 - 3(3)^2 - 11(3) + 6 = 54 - 27 - 33 + 6 = 0$

3 (c) (ii)

> **STEPS**
> 1. Guess at a root (unless a root is given) by substituting in numbers $0, 1, -1, 2, -2, ...$ until you get zero.
> 2. Using the factor theorem, form a factor from the root.
> 3. Divide the cubic by the factor to get the quadratic.
> 4. Solve the quadratic by factorising or using formula **2**.
> 5. Write down the three roots.

1. $f(3) = 0 \Rightarrow 3$ is a root.

2. $\therefore (x-3)$ is a factor.

3. Divide the factor into the cubic as shown on the right.

$$\begin{array}{r|l} & 2x^2 + 3x - 2 \\ \hline x - 3 & 2x^3 - 3x^2 - 11x + 6 \\ & \mp 2x^3 \pm 6x^2 \\ \hline & \quad 3x^2 - 11x + 6 \\ & \quad \mp 3x^2 \pm 9x \\ \hline & \quad\quad -2x + 6 \\ & \quad\quad \pm 2x \mp 6 \\ \hline & \quad\quad\quad 0 \end{array}$$

$\therefore 2x^3 - 3x^2 - 11x + 6 = (x-3)(2x^2 + 3x - 2) = 0$

4. Factorise the quadratic.

$2x^2 + 3x - 2 = (2x-1)(x+2)$

5. Write down the three roots.

$\therefore 2x^3 - 3x^2 - 11x + 6 = (x-3)(2x-1)(x+2) = 0$

$\therefore x = -2, \frac{1}{2}, 3$