ALGEBRA (Q 2 & 3, PAPER 1)

2004

2 (a) Find the value of $3(2p-q)$ when $p=-4$ and $q=5$.

(b) (i) Solve $2x^2-7x+3=0$.

(ii) Show that $x-2$ is a factor of x^3-3x^2-x+6.

(c) (i) Evaluate $8^{\frac{1}{3}}$.

(ii) Express $4^{\frac{1}{4}}$ in the form 2^k, $k\in\mathbf{Q}$.

(iii) Solve for x the equation

$$(8^{\frac{1}{3}})(4^{\frac{1}{4}})=2^{5-x}.$$

SOLUTION

2 (a)

$3(2p-q)=3(2(-4)-5))=3(-8-5)=3(-13)=-39$

2 (b) (i)

$2x^2-7x+3=0$

$\Rightarrow(2x-1)(x-3)=0$ [Set each factor equal to zero and solve for x.]

$\therefore(2x-1)=0\Rightarrow 2x=1\Rightarrow x=\frac{1}{2}$

$\therefore(x-3)=0\Rightarrow x=3$

2 (b) (ii)

The factor theorem states that:

> If $(x-k)$ is a factor of $f(x)$ then k is a root of $f(x)=0$,
> i.e. $f(k)=0$ and vice versa.

If $x-2$ is a factor $\Rightarrow 2$ is a root. Substitute 2 in for x.

$f(x)=x^3-3x^2-x+6\Rightarrow f(2)=(2)^3-3(2)^2-(2)+6$

$=8-12-2+6=0$

Because $f(2)=0\Rightarrow 2$ is a root. Therefore, $x-2$ is a factor.

2 (c) (i)

$8^{\frac{1}{3}}=2$

> The third root of 8 is the number multiplied by itself three times to give 8.

2 (c) (ii)

$4^{\frac{1}{4}}=(2^2)^{\frac{1}{4}}=2^{\frac{1}{2}}$

> POWER RULES
> 5. $(a^m)^n=a^{mn}$

2 (c) (iii)

STEPS
1. Tidy up both sides using the Power Rules until you have the same base and nothing else on both sides.
2. Put the powers equal to one another.
3. Solve for the variable (usually *x*.)

1. $(8^{\frac{1}{3}})(4^{\frac{1}{4}})=2^{5-x} \Rightarrow (2^1)(2^{\frac{1}{2}})=2^{5-x}$

$\Rightarrow 2^{\frac{3}{2}}=2^{5-x}$

POWER RULES
1. $a^m \times a^n = a^{m+n}$

2. $\therefore \frac{3}{2}=5-x$

3. $\frac{3}{2}=5-x \Rightarrow 3=10-2x$ [Multiplied across by 2.]

$\Rightarrow 2x=10-3 \Rightarrow 2x=7$

$\therefore x=\frac{7}{2}$

3 (a) Solve for *x*

$$2x=3(5-x).$$

(b) Solve for *x* and *y*

$$x+y=1$$
$$x^2+y^2=13.$$

(c) *p* is a positive number and *f* is the function $f(x)=(2x+p)(x-p),\ x\in\mathbf{R}$.

(i) Given that $f(2)=0$, find the value of *p*.

(ii) Hence, find the range of values of *x* for which $f(x)<0$.

SOLUTION

3 (a)

$2x=3(5-x) \Rightarrow 2x=15-3x$

$\Rightarrow 2x+3x=15$

$\Rightarrow 5x=15$

$\Rightarrow x=\frac{15}{5}=3$

Multiply every term by every term and then tidy up by adding and subtracting like terms.

3 (b)

STEPS
1. Eliminate a letter from the linear equation.
2. Substitute into quadratic and solve for the other letter.
3. Substitute these values into the linear to get all solutions.

1. Isolate the x.

$x+y=1$
$\Rightarrow x=1-y ...(\mathbf{A})$

→

2.

$x^2+y^2=1$
$\Rightarrow (1-y)^2+y^2=13$
$\Rightarrow 1-2y+y^2+y^2=13$
$\Rightarrow 2y^2-2y-12=0$
$\Rightarrow y^2-y-6=0$
$\Rightarrow (y-3)(y+2)=0$
$\therefore y=-2, 3$

←

3. Substitute the values of y into Eqn. (**A**) to obtain the x values.

$y=-2: x=1-(-2)=3$
$y=3: x=1-(3)=2$

ANSWER: (3, −2), (−2, 3)

3 (c) (i)

$f(x)=(2x+p)(x-p) \Rightarrow f(2)=(4+p)(2-p)=0$

Set each bracket equal to zero and solve for p.

$\therefore p=-4, 2$

As $p > 0$, ignore the negative solution.

$\therefore p=2$

3 (c) (ii)

$\therefore f(x)=(2x+2)(x-2)=2x^2-2x-4<0$

Solving quadratic inequalities:

STEPS
1. Find the roots of the quadratic equation: $ax^2+bx+c=0$. These are the places where the curve crosses the x-axis.
2. Sketch the graph. It is either ∪ shaped or ∩ shaped.
3. Use the graph to solve the inequality.
   $y=f(x)>0$ is above the x-axis.
   $y=f(x)<0$ is below the x-axis.

1. Solve $2x^2-2x-4=0 \Rightarrow x^2-x-2=0 \Rightarrow (x-2)(x+1)=0$
   $\therefore x=-1, 2$
2. Sketch the graph. The coefficient of x^2 is positive so the graph is ∪ shaped.
3. You can see the parts of the graph that are less than zero, i.e. below the x-axis.