ALGEBRA (Q 2 & 3, PAPER 1)

2003

2 (a) Given that $3x-2y=4$, find the value of y when $x=-2$.

(b) (i) Evaluate $9^{\frac{1}{2}}$.

(ii) Express $\sqrt{8}$ in the form 2^k, $k \in \mathbf{Q}$.

(iii) Solve for x the equation $25^x = 5^{6-x}$.

(c) Solve for x the equation

$$\frac{3}{x+1}+\frac{1}{x-1}=1.$$

Give your answers in the form $a \pm \sqrt{b}$, where $a, b \in \mathbf{N}$.

SOLUTION

2 (a)

$3x-2y=4 \Rightarrow 3(-2)-2y=4$

$\Rightarrow -6-2y=4 \Rightarrow -6-4=2y$

$\Rightarrow -10=2y \Rightarrow y=-5$

2 (b) (i)

$9^{\frac{1}{2}}=3$

What number multiplied by itself gives 9?

2 (b) (ii)

$\sqrt{8}=8^{\frac{1}{2}}=(2^3)^{\frac{1}{2}}=2^{\frac{3}{2}}$

POWER RULES

5. $(a^m)^n = a^{mn}$ **Ex.** $(x^3)^2 = x^6$

6. $\sqrt{a}=a^{\frac{1}{2}}$ **Ex.** $\sqrt{9}=9^{\frac{1}{2}}=3$

2 (b) (iii)

STEPS

1. Tidy up both sides using the Power Rules until you have the same base and nothing else on both sides.
2. Put the powers equal to one another.
3. Solve for the variable (usually x.)

1. $25^x=5^{6-x} \Rightarrow (5^2)^x=5^{6-x}$

$\Rightarrow 5^{2x}=5^{6-x}$

2. $\therefore 2x=6-x$

3. $\Rightarrow 2x+x=6 \Rightarrow 3x=6$

$\therefore x=2$

2 (c)

$\frac{3}{x+1}+\frac{1}{x-1}=1$ [Multiply each term by $(x+1)(x-1)$.]

$\Rightarrow \frac{3(x+1)(x-1)}{(x+1)}+\frac{1(x+1)(x-1)}{(x-1)}=1(x+1)(x-1)$

$\Rightarrow 3(x-1)+(x+1)=(x+1)(x-1)$ [Multiply out the brackets.]

$\Rightarrow 3x-3+x+1=x^2-1$

$\Rightarrow x^2-4x+1=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **2**

$a=1$
$b=-4$
$c=1$

$\therefore x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}=\frac{4\pm\sqrt{16-4}}{2}=\frac{4\pm\sqrt{12}}{2}$

$=\frac{4\pm2\sqrt{3}}{2}=2\pm\sqrt{3}$

Note: $\sqrt{12}=\sqrt{4\times3}=2\sqrt{3}$

3 (a) Find the solution set of
$5x-3<12,\ x\in\mathbf{N}.$

(b) (i) Show that $x + 2$ is a factor of $x^3+3x^2-4x-12$.

(ii) Hence, or otherwise, solve the equation $x^3+3x^2-4x-12=0$.

(c) (i) Simplify $(x+\sqrt{a-x})(x-\sqrt{a-x})$, where $a-x\geq 0$.

(ii) Given that $x = 3$ is a solution of the equation $(x+\sqrt{a-x})(x-\sqrt{a-x})=0$, find the value of a.

(iii) Hence, find the other solution of the equation in part **(ii)**, and verify your answer.

Solution

3 (a)

$5x-3<12\Rightarrow 5x<12+3$

$\Rightarrow 5x<15\Rightarrow x<3$

$\therefore x=\{0, 1, 2\}$

N: Set of natural numbers. These are whole positive numbers.
N = {0, 1, 2, 3,...}

3 (b) (i)

If $(x + 2)$ is a factor $\Rightarrow -2$ is a root. Substitute -2 in for x.

$f(x)=x^3+3x^2-4x-12\Rightarrow f(-2)=(-2)^3+3(-2)^2-4(-2)-12$

$=-8+12+8-12=0$

Because $f(-2)=0\Rightarrow -2$ is a root. Therefore, $(x + 2)$ is a factor.

3 (b) (ii)

Steps
1. Guess at a root (unless a root is given) by substituting in numbers $0, 1, -1, 2, -2, ...$ until you get zero.
2. Using the factor theorem, form a factor from the root.
3. Divide the cubic by the factor to get the quadratic.
4. Solve the quadratic by factorising or using formula **2**.
5. Write down the three roots.

Steps 1 and 2 already done in 3 (b) (i).

3. Divide the cubic by the factor as shown to the right.

$\therefore x^3+3x^2-4x-12=(x+2)(x^2+x-6)=0$

$$
\begin{array}{r|l}
 & x^2+x-6 \\
\hline
x+2 & x^3+3x^2-4x-12 \\
 & \mp x^3 \mp 2x^2 \\
 & \quad x^2-4x-12 \\
 & \quad \mp x^2 \mp 2x \\
 & \quad\quad -6x-12 \\
 & \quad\quad \pm 6x \pm 12 \\
 & \quad\quad\quad 0
\end{array}
$$

4. The resulting quadratic can be factorised.

$x^2+x-6=(x+3)(x-2)$

5. $\therefore x^3+3x^2-4x-12=(x+2)(x+3)(x-2)=0$

Set each factor equal to zero and solve for x.

$\therefore x=-3, -2, 2$

3 (c) (i)

$(x+\sqrt{a-x})(x-\sqrt{a-x})=x^2-x\sqrt{a-x}+x\sqrt{a-x}-\sqrt{a-x}\sqrt{a-x}$

$=x^2-(a-x)=x^2+x-a$

3 (c) (ii)

$(x+\sqrt{a-x})(x-\sqrt{a-x})=0\Rightarrow x^2+x-a=0$

$x=3\Rightarrow 3^2+3-a=0\Rightarrow 9+3-a=0\Rightarrow a=12$

3 (c) (iii)

$x^2+x-12=0\Rightarrow(x+4)(x-3)=0$

$\therefore x=-4, 3$

Therefore, the other solution is $x=-4$.