ALGEBRA (Q 2 & 3, PAPER 1)

2002

2 (a) Solve for x: $\frac{x-7}{2}=\frac{x+3}{6}$.

(b) (i) Show that $x + 2$ is a factor of $2x^3+7x^2+x-10$.

(ii) Hence, or otherwise, find the three roots of $2x^3+7x^2+x-10=0$.

(c) (i) Express b in terms of a and c where $\frac{8a-5b}{b}=c$.

(ii) Hence, or otherwise, evaluate b when $a=2^{\frac{5}{2}}$ and $c = 3^3$.

SOLUTION

2 (a)

$\frac{x-7}{2}=\frac{x+3}{6}$ [Multiply each side by 6.]

$\Rightarrow \frac{6(x-7)}{2}=\frac{6(x+3)}{6} \Rightarrow 3(x-7)=1(x+3)$

$\Rightarrow 3x-21=x+3$

$\Rightarrow 3x-x=3+21$

$\Rightarrow 2x=24 \Rightarrow x=12$

2 (b) (i)

The factor theorem states that:

If $(x-k)$ is a factor of $f(x)$ then k is a root of $f(x) = 0$, i.e. $f(k) = 0$ and vice versa.

If $(x + 1)$ is a factor $\Rightarrow -1$ is a root. Substitute -1 in for x.

$f(x)=x^3-2x^2+7x+10 \Rightarrow f(-1)=(-1)^3-2(-1)^2+7(-1)+10$

$=-1-2(1)-7+10=-1-2-7+10=0$

Because $f(-1)=0 \Rightarrow -1$ is a root. Therefore, $(x + 1)$ is a factor.

2 (b) (ii)

STEPS
1. Guess at a root (unless a root is given) by substituting in numbers $0, 1, -1, 2, -2, ...$ until you get zero.
2. Using the factor theorem, form a factor from the root.
3. Divide the cubic by the factor to get the quadratic.
4. Solve the quadratic by factorising or using formula **2**.
5. Write down the three roots.

Steps 1 and 2 are already done in 2 (b) (i).

3. Divide the factor into the cubic.

$$\therefore 2x^3+7x^2+x-10=(x+2)(2x^2+3x-5)=0$$

$$
\begin{array}{r|l}
 & 2x^2+3x-5 \\
x+2 & 2x^3+7x^2+x-10 \\
 & \mp 2x^3 \mp 4x^2 \\
 & \quad 3x^2+x-10 \\
 & \quad \mp 3x^2 \mp 6x \\
 & \qquad -5x-10 \\
 & \qquad \pm 5x \pm 10 \\
 & \qquad 0
\end{array}
$$

4. $2x^2+3x-5=(2x+5)(x-1)$

5. $\therefore 2x^3+7x^2+x-10=(x+2)(2x+5)(x-1)=0$

Set each factor equal to zero and solve for x.

$\therefore x=-2, -\frac{5}{2}, 1$

2 (c) (i)

$\frac{8a-5b}{b}=c$ [Multiply both sides by b.]

$\Rightarrow 8a-5b=bc$ [Bring the b terms to the right.]

$\Rightarrow 8a=bc+5b$ [Factorise the right hand side by taking out b in common.]

$\Rightarrow 8a=b(c+5)$ [Divide both sides by $(c+5)$.]

$$\Rightarrow \frac{8a}{(c+5)}=b$$

2 (c) (ii)

$\Rightarrow b=\frac{8a}{(c+5)} \Rightarrow b=\frac{8(2^{\frac{5}{2}})}{(3^3+5)}$ [**NOTE**: $3^3=27$]

$\Rightarrow b=\frac{8(2^{\frac{5}{2}})}{(27+5)}=\frac{8(2^{\frac{5}{2}})}{32}$ [Change each number into a power of 2.]

$\Rightarrow b=\frac{(2^3)(2^{\frac{5}{2}})}{2^5}$ [Use the power rules]

$\Rightarrow b=\frac{2^{\frac{11}{2}}}{2^5}=2^{\frac{1}{2}}=\sqrt{2}$

POWER RULES

1. $a^m \times a^n = a^{m+n}$	**Ex.** $x^3 \times x^2 = x^5$
2. $\frac{a^m}{a^n}=a^{m-n}$	**Ex.** $\frac{x^5}{x^3}=x^2$
6. $\sqrt{a}=a^{\frac{1}{2}}$	**Ex.** $\sqrt{9}=9^{\frac{1}{2}}=3$

3 (a) Solve the inequality $5x+1 \geq 4x-3,\ x \in \mathbf{R}$ and illustrate the solution set on a number line.

(b) (i) Solve for x and y

$$y = 10 - 2x$$
$$x^2 + y^2 = 25.$$

(ii) Hence, find the two possible values of $x^3 + y^3$.

(c) Let $f(x) = x^2 + ax + t$ where $a,\ t \in \mathbf{R}$.

(i) Find the value of a, given that $f(-5) = f(-1)$.

(ii) Given that there is only one value of x for which the $f(x) = 0$, find the value of t.

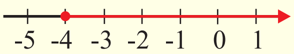
SOLUTION

3 (a)

$5x+1 \geq 4x-3 \Rightarrow 5x-4x \geq -3-1$

$\Rightarrow x \geq -4$

R: Set of real numbers. This set represents all numbers.



3 (b) (i)

STEPS
1. Eliminate a letter from the linear equation.
2. Substitute into quadratic and solve for the other letter.
3. Substitute these values into the linear to get all solutions.

1. $y = 10 - 2x \ldots(\mathbf{A})$ →

2.
$x^2 + y^2 = 25$
$\Rightarrow x^2 + (10-2x)^2 = 25$
$\Rightarrow x^2 + 100 - 40x + 4x^2 = 25$
$\Rightarrow 5x^2 - 40x + 75 = 0$
$\Rightarrow x^2 - 8x + 15 = 0$
$\Rightarrow (x-5)(x-3) = 0$
$\Rightarrow x = 3,\ 5$

3. Substitute the values of x into Eqn. (**A**) to obtain the y values.

←
$x = 3$: $y = 10 - 2(3) = 4$
$x = 5$: $y = 10 - 2(5) = 0$

ANSWER: (3, 4), (5, 0)

3 (b) (ii)

$x = 3, y = 4$: $\therefore x^3 + y^3 = 3^3 + 4^3 = 27 + 64 = 91$

$x = 5, y = 0$: $\therefore x^3 + y^3 = 5^3 + 0^3 = 125 + 0 = 125$

3 (c) $f(x) = x^2 + ax + t$

3 (c) (i)

$f(-5) = f(-1) \Rightarrow (-5)^2 + a(-5) + t = (-1)^2 + a(-1) + t$

$\Rightarrow 25 - 5a + t = 1 - a + t$

$\Rightarrow 25 - 1 = -a + 5a$

$\Rightarrow 24 = 4a \Rightarrow a = 6$

3 (c) (ii)

$f(x) = 0 \Rightarrow x^2 + 6x + t = 0$

There is only one value of *x* for which the *f*(*x*) = 0 means that the quadratic equations has equal roots.

The quadratic equation $ax^2 + bx + c = 0$ has equal roots if $b^2 = 4ac$.

$a = 1$
$b = 6$
$c = t$

$b^2 = 4ac \Rightarrow 6^2 = 4(1)(t) \Rightarrow 36 = 4t$

$\therefore t = 9$