ALGEBRA (Q 2 & 3, PAPER 1)

2001

2 (a) Find the solution set of $11-2n>3,\ n\in \mathbf{N}$.

(b) Solve for x and y

$x+2y=3$

$x^2-y^2=24.$

(c) Solve each of the following equations for p

(i) $9^p=\dfrac{1}{\sqrt{3}}$

(ii) $2^{3p-7}=2^6-2^5.$

SOLUTION

2 (a)

$11-2n>3 \Rightarrow -2n>3-11$

$\Rightarrow -2n>-8$ [Divide both sides by -2. Remember to reverse the inequality.]

$\Rightarrow n<4$

$\therefore n=\{0,1,2,3\}$

N: Set of natural numbers. These are whole positive numbers.
N = {0, 1, 2, 3,...}

2 (b)

STEPS
1. Eliminate a letter from the linear equation.
2. Substitute into quadratic and solve for the other letter.
3. Substitute these values into the linear to get all solutions.

1. Isolate the x as this is the easier letter to get on its own.

$x+2y=3$

$\Rightarrow x=3-2y....(\mathbf{A})$

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2.

$x^2-y^2=24$

$\Rightarrow (3-2y)^2-y^2=24$

$\Rightarrow 9-12y+4y^2-y^2=24$

$\Rightarrow 3y^2-12y-15=0$

$\Rightarrow y^2-4y-5=0$

$\Rightarrow (y-5)(y+1)=0$

$\Rightarrow y=5,\ -1$

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3. Substitute this value of y into Eqn. (**A**) to obtain the x value.

$y=5: x=3-2(5)=-7$

$y=-1: x=3-2(-1)=5$

ANSWER: $(-7,5),\ (5,-1)$

2 (c) (i)

$9^p = \frac{1}{\sqrt{3}}$ [Change everything to base 3.]

$\Rightarrow (3^2)^p = \frac{1}{3^{\frac{1}{2}}}$ [Use the power rules.]

$\Rightarrow 3^{2p} = 3^{-\frac{1}{2}}$ [Equate the powers.]

$\Rightarrow 2p = -\frac{1}{2}$

$\therefore p = -\frac{1}{4}$

POWER RULES

1. $a^m \times a^n = a^{m+n}$	**Ex.** $x^3 \times x^2 = x^5$
2. $\frac{a^m}{a^n} = a^{m-n}$	**Ex.** $\frac{x^5}{x^3} = x^2$
3. $a^0 = 1$	**Ex.** $5^0 = 1$
4. $a^{-n} = \frac{1}{a^n}$	**Ex.** $x^{-3} = \frac{1}{x^3}$
5. $(a^m)^n = a^{mn}$	**Ex.** $(x^3)^2 = x^6$
6. $\sqrt{a} = a^{\frac{1}{2}}$	**Ex.** $\sqrt{9} = 9^{\frac{1}{2}} = 3$

2 (c) (ii)

$2^{3p-7} = 2^6 - 2^5$ [Work out the right hand side by calculating each number.]

$\Rightarrow 2^{3p-7} = 64 - 32 \Rightarrow 2^{3p-7} = 32$ [Change everything to base 2.]

$\Rightarrow 2^{3p-7} = 2^5$ [Equate the powers.]

$\Rightarrow 3p - 7 = 5 \Rightarrow 3p = 12$

$\therefore p = 4$

3 (a) Given that $u^2 + 2as = v^2$, calculate the value of a when u = 10, s = 30 and v = 20.

(b) (i) Simplify $(x+\sqrt{x})(x-\sqrt{x})$ when $x > 0$.

(ii) Hence, or otherwise, find the value of x for which $(x+\sqrt{x})(x-\sqrt{x}) = 6$.

(c) Let $f(x) = x^3 + ax^2 + bx - 6$ where a and b are real numbers.
Given that $x-1$ and $x-2$ are factors of $f(x)$

(i) find the value of a and the value of b

(ii) hence, find the values of x for which $f(x) = 0$.

SOLUTION

3 (a)

$u^2 + 2as = v^2 \Rightarrow (10)^2 + 2a(30) = 20^2$

$\Rightarrow 100 + 60a = 400 \Rightarrow 60a = 300$

$\Rightarrow a = \frac{300}{60} \Rightarrow a = 5$

3 (b) (i)

$(x+\sqrt{x})(x-\sqrt{x}) = x^2 - x\sqrt{x} + x\sqrt{x} - \sqrt{x}\sqrt{x}$

$= x^2 - x$

3 (b) (ii)

$(x+\sqrt{x})(x-\sqrt{x}) = 6 \Rightarrow x^2 - x = 6$

$\Rightarrow x^2 - x - 6 = 0$ [Factorise the quadratic.]

$\Rightarrow (x-3)(x+2) = 0$ [Set each factor equal to zero and solve for *x*.]

$\therefore x = -2,\ 3$

3 (c)

The factor theorem states that:

If $(x-k)$ is a factor of $f(x)$ then k is a root of $f(x) = 0$, i.e. $f(k) = 0$ and vice versa.

3 (c) (i)

If $x-1$ is a factor of $f(x)$, then $f(1) = 0$.

$\therefore\ f(1) = (1)^3 + a(1)^2 + b(1) - 6 = 0$

$\Rightarrow 1 + a + b - 6 = 0 \Rightarrow a + b = 5$.....(**1**)

If $x-2$ is a factor of $f(x)$, then $f(2) = 0$.

$\therefore\ f(2) = (2)^3 + a(2)^2 + b(2) - 6 = 0$

$\Rightarrow 1 + a + b - 6 = 0 \Rightarrow a + b = 5$.....(**1**)

$\Rightarrow 8 + 4a + 2b - 6 = 0 \Rightarrow 4a + 2b = -2$

$\Rightarrow 2a + b = -1$.....(**2**)

Solve equations (**1**) and (**2**) simultaneously.

$a + b = 5$........(**1**)
$2a + b = -1$.....(**2**)$(\times -1)$

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$$\begin{array}{rl} a + b &= 5 \\ -2a - b &= 1 \\ \hline -a \quad\ &= 6 \Rightarrow a = -6 \end{array}$$

Substitute this value of a into Eqn. (1) $\Rightarrow -6 + b = 5 \Rightarrow b = 11$

Answer: $a = -6,\ b = 11$

3 (c) (ii)

$f(x) = 0 \Rightarrow x^3 - 6x^2 + 11x - 6 = 0$

The 2 linear factors multiply to give a quadratic.

$(x-1)(x-2) = x^2 - 3x + 2$

Divide this quadratic into the cubic to get the other linear factor.

$$\begin{array}{r|l} & x - 3 \\ \hline x^2 - 3x + 2 & x^3 - 6x^2 + 11x - 6 \\ & \mp x^3 \pm 3x^2 \mp 2x \\ \hline & \quad -3x^2 + 9x - 6 \\ & \quad \pm 3x^2 \mp 9x \pm 6 \\ \hline & \qquad 0 \end{array}$$

$\therefore x^3 - 6x^2 + 11x - 6 = (x-1)(x-2)(x-3) = 0$

Set each factor equal to zero and solve for *x*.

$\therefore x = 1,\ 2,\ 3$