ALGEBRA (Q 2 & 3, PAPER 1)

2000

2 (a) Find the value of $5x-3y$ when $x=\frac{5}{2}$ and $y=\frac{2}{3}$.

(b) Solve for x and y

$x-3y=1$

$x^2-y^2=0.$

(c) Write as a power of 3

(i) 243

(ii) $\sqrt{27}$.

Hence, solve for x the equation $\sqrt{3}(3^x)=\left(\frac{243}{\sqrt{27}}\right)^2$.

SOLUTION

2 (a)

$5x-3y=5(\frac{5}{2})-3(\frac{2}{3})=\frac{25}{2}-2=\frac{21}{2}$

2 (b)

STEPS
1. Eliminate a letter from the linear equation.
2. Substitute into quadratic and solve for the other letter.
3. Substitute these values into the linear to get all solutions.

1. Isolate the x as this is the easier letter to get on its own.

$x-3y=1$
$\Rightarrow x=3y+1 ...(\mathbf{A})$

2.

$\Rightarrow x^2-y^2=0$
$\Rightarrow (3y+1)^2-y^2=0$
$\Rightarrow 9y^2+6y+1-y^2=0$
$\Rightarrow 8y^2+6y+1=0$
$\Rightarrow (4y+1)(2y+1)=0$
$\therefore y=-\frac{1}{4}, -\frac{1}{2}$

3. Substitute this value of y into Eqn. (**A**) to obtain the x value.

$y=-\frac{1}{4}: x=3(-\frac{1}{4})+1=-\frac{3}{4}+1=\frac{1}{4}$
$y=-\frac{1}{2}: x=3(-\frac{1}{2})+1=-\frac{3}{2}+1=-\frac{1}{2}$

ANSWER: $(\frac{1}{4}, -\frac{1}{4}), (-\frac{1}{2}, -\frac{1}{2})$

2 (c) (i)

$243 = 3^5$

2 (c) (ii)

$\sqrt{27} = (27)^{\frac{1}{2}} = (3^3)^{\frac{1}{2}} = 3^{\frac{3}{2}}$

$\sqrt{3}(3^x) = \left(\frac{243}{\sqrt{27}}\right)^2 \Rightarrow 3^{\frac{1}{2}} \times 3^x = \left(\frac{3^5}{3^{\frac{3}{2}}}\right)^2$

$\Rightarrow 3^{x+\frac{1}{2}} = (3^{\frac{7}{2}})^2$

$\Rightarrow 3^{x+\frac{1}{2}} = 3^7$

$\Rightarrow x + \frac{1}{2} = 7 \Rightarrow x = 7 - \frac{1}{2}$

$\therefore x = \frac{13}{2}$

Power Rules

1. $a^m \times a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $a^0 = 1$
4. $a^{-n} = \frac{1}{a^n}$
5. $(a^m)^n = a^{mn}$
6. $\sqrt{a} = a^{\frac{1}{2}}$

3 (a) Express p in terms of t and k when
$tp - k = 7k,\ t \neq 0.$

(b) (i) Show that $x = 2$ is a root of $3x^3 + 8x^2 - 33x + 10 = 0.$

(ii) Find the other roots of $3x^3 + 8x^2 - 33x + 10 = 0.$

(c) (i) $f(x) = ax^2 + bx - 8$, where a and b are real numbers.
If $f(1) = -9$ and $f(-1) = 3$, find the value of a and the value of b.

(ii) Using your values of a and b from **(i)**, find the two values of x for which
$ax^2 + bx = bx^2 + ax.$

Solution

3 (a)

$tp - k = 7k \Rightarrow tp = 7k + k$ [Bring the k terms to the right and add them together.]

$\Rightarrow tp = 8k$ [Divide both sides by t.]

$\Rightarrow p = \frac{8k}{t}$

3 (b) (i)

$f(x) = 3x^3 + 8x^2 - 33x + 10 \Rightarrow f(2) = 3(2)^3 + 8(2)^2 - 33(2) + 10$

$= 24 + 32 - 66 + 10 = 0$

3 (b) (ii)

STEPS TO SOLVING A CUBIC
1. Find a root by guessing.
2. Get a factor from the root.
3. Divide the factor into the cubic.
4. Factorise the resulting quadratic.
5. Write down the three roots.

1. $f(2) = 0$
2. $\therefore (x-2)$ is a factor.
3. Divide this factor into the cubic as shown to the right.

   $\therefore 3x^3 + 8x^2 - 33x + 10 = (x-2)(3x^2 + 14x - 5) = 0$

$$
\begin{array}{r|l}
 & 3x^2 + 14x - 5 \\
\hline
x - 2 & 3x^3 + 8x^2 - 33x + 10 \\
 & \mp 3x^3 \pm 6x^2 \\
\hline
 & 14x^2 - 33x + 10 \\
 & \mp 14x^2 \pm 28x \\
\hline
 & -5x + 10 \\
 & \pm 5x \mp 10 \\
\hline
 & 0
\end{array}
$$

4. Factorise the quadratic.

   $3x^2 + 14x - 5 = (3x-1)(x+5)$

5. $\therefore 3x^3 + 8x^2 - 33x + 10 = (x-2)(3x-1)(x+5) = 0$

   Set each factor equal to zero and solve for x.

   $\therefore x = -5, \frac{1}{3}, 2$

3 (c) (i)

$f(x) = ax^2 + bx - 8$

$f(1) = -9 \Rightarrow a(1)^2 + b(1) - 8 = -9 \Rightarrow a + b = -1....(\mathbf{1})$

$f(-1) = 3 \Rightarrow a(-1)^2 + b(-1) - 8 = 3 \Rightarrow a - b = 11....(\mathbf{2})$

Solve equations (**1**) and (**2**) simultaneously.

$$
\begin{array}{l}
a + b = -1...(\mathbf{1}) \\
a - b = 11....(\mathbf{2}) \\
\hline
2a \quad = 10 \Rightarrow a = 5
\end{array}
$$

Substitute the value for a back into Eqn. (**1**) $\Rightarrow 5 + b = -1 \Rightarrow b = -6$

3 (c) (ii)

$ax^2 + bx = bx^2 + ax \Rightarrow 5x^2 - 6x = -6x^2 + 5x$ [Bring all terms to the left.]

$\Rightarrow 11x^2 - 11x = 0$ [Factorise the quadratic.]

$\Rightarrow 11x(x-1) = 0$ [Set each factor equal to zero and solve for x.]

$\therefore x = 0, 1$