ALGEBRA (Q 2 & 3, PAPER 1)

1999

2 (a) Solve for x

$$2(x+8)=7x.$$

(b) Write as a power of 2

(i) 8

(ii) $8^{\frac{4}{3}}$.

Solve for x the equation

$$8^{\frac{4}{3}}=\frac{2^{5x-4}}{\sqrt{2}}.$$

(c) Solve for x

$$\frac{3}{2x-1}=1+\frac{2x}{x+2},\ x\neq\tfrac{1}{2} \text{ and } x\neq -2.$$

SOLUTION

2 (a)

$2(x+8)=7x$ [Multiply out the brackets.]

$\Rightarrow 2x+16=7x$ [Bring the x terms tothe right.]

$\Rightarrow 16=7x-2x$ [Add the x terms.]

$\Rightarrow 16=5x$ [Isolate the x by dividing both sides by 5.]

$\therefore x=\frac{16}{5}$

2 (b) (i)

$8=2^3$

2 (b) (ii)

$8^{\frac{4}{3}}=(2^3)^{\frac{4}{3}}=2^4$

POWER RULES	
1. $a^m\times a^n=a^{m+n}$	**Ex.** $x^3\times x^2=x^5$
2. $\frac{a^m}{a^n}=a^{m-n}$	**Ex.** $\frac{x^5}{x^3}=x^2$
3. $a^0=1$	**Ex.** $5^0=1$
4. $a^{-n}=\frac{1}{a^n}$	**Ex.** $x^{-3}=\frac{1}{x^3}$
5. $(a^m)^n=a^{mn}$	**Ex.** $(x^3)^2=x^6$
6. $\sqrt{a}=a^{\frac{1}{2}}$	**Ex.** $\sqrt{9}=9^{\frac{1}{2}}=3$

1. $8^{\frac{4}{3}}=\frac{2^{5x-4}}{\sqrt{2}}\Rightarrow 2^4=\frac{2^{5x-4}}{2^{\frac{1}{2}}}$

$\Rightarrow 2^4=2^{5x-\frac{9}{2}}$

2. $\Rightarrow 4=5x-\frac{9}{2}$

3. $\Rightarrow 4+\frac{9}{2}=5x\Rightarrow \frac{17}{2}=5x$

$\therefore x=\frac{17}{10}$

STEPS
1. Tidy up both sides using the Power Rules until you have the same base and nothing else on both sides.
2. Put the powers equal to one another.
3. Solve for the variable (usually x.)

2 (c)

$\frac{3}{2x-1}=1+\frac{2x}{x+2}$ [Multiply each term by $(2x-1)(x+2)$.]

$\Rightarrow \frac{3(2x-1)(x+2)}{(2x-1)}=1(2x-1)(x+2)+\frac{2x(2x-1)(x+2)}{(x+2)}$ [Cancel brackets that are the same.]

$\Rightarrow 3(x+2)=1(2x-1)(x+2)+2x(2x-1)$ [Multiply out the brackets.]

$\Rightarrow 3x+6=2x^2+3x-2+4x^2-2x$ [Bring all the terms to the right.]

$\Rightarrow 0=6x^2-2x-8$ [Divide across by 2.]

$\Rightarrow 3x^2-x-4=0$ [Factorise the quadratic.]

$\Rightarrow (3x-4)(x+1)=0$ [Set eqch factor equal to zero and solve for x.]

$\therefore x=-1, \frac{4}{3}$

3 (a) Express p in terms of q and r when

$$\frac{p-3r}{q}=5,\ q\neq 0.$$

(b) Solve for x and y

$$x+2y=6$$
$$x^2+y^2=17.$$

(c) Show that $6x^2+5x-4$ is a factor of $6x^3+17x^2+6x-8$.

Hence, or otherwise, find the roots of $6x^3+17x^2+6x-8=0$.

SOLUTION

3 (a)

$\frac{p-3r}{q}=5$ [Multiply both sides by q.]

$\Rightarrow p-3r=5q$

$\Rightarrow p=5q+3r$

3 (b)

STEPS
1. Eliminate a letter from the linear equation.
2. Substitute into quadratic and solve for the other letter.
3. Substitute these values into the linear to get all solutions.

1. Isolate the x as this is the easier letter to get on its own.

$$x+2y=6$$
$$\Rightarrow x=6-2y....(\mathbf{A})$$

2.

$$x^2+y^2=17$$
$$\Rightarrow (6-2y)^2+y^2=17$$
$$\Rightarrow 36-24y+4y^2+y^2=17$$
$$\Rightarrow 5y^2-24y+19=0$$
$$\Rightarrow (5y-19)(y-1)=0$$
$$\therefore y=\tfrac{19}{5}, 1$$

3. Substitute this value of y into Eqn. (**A**) to obtain the x value.

$$y=1: x=6-2(1)=6-2=4$$
$$y=\tfrac{19}{5}: x=6-2(\tfrac{19}{5})=6-\tfrac{38}{5}=-\tfrac{8}{5}$$

ANSWER: $(-\frac{8}{5}, \frac{19}{5}), (4, 1)$

3 (c)

To show $6x^2+5x-4$ is a factor of $6x^3+17x^2+6x-8$, divide it in and show the remainder is zero.

$$
\begin{array}{r|l}
 & \quad x+2 \\
6x^2+5x-4 & \; 6x^3+17x^2+6x-8 \\
 & \mp 6x^3 \mp 5x^2 \pm 4x \\
 & \qquad\quad 12x^2+10x-8 \\
 & \qquad\;\; \mp 12x^2 \mp 10x \pm 8 \\
 & \qquad\qquad\qquad 0
\end{array}
$$

$\therefore 6x^3+17x^2+6x-8=(6x^2+5x-4)(x+2)=0$

Factorise the quadratic: $6x^2+5x-4=(3x+4)(2x-1)$

$\therefore 6x^3+17x^2+6x-8=(3x+4)(2x-1)(x+2)=0$

Set each factor is equal to zero and solve for x.

$\therefore x=-\frac{4}{3}, \frac{1}{2}, -2$