ALGEBRA (Q 2 & 3, PAPER 1)

1998

2 (a) Solve

$$5x - 2y = 13$$
$$3(x-4) = 4y.$$

(b) Find the value of

$$\frac{a-b+1}{a+b+1}$$

when $a = \frac{1}{8}$ and $b = 2$.

(c) (i) Write $\sqrt{125}$ as a power of 5.

(ii) Solve for x the equation

$$\frac{5^{2x+1}}{\sqrt{5}} = \left(\frac{1}{\sqrt{125}}\right)^3.$$

SOLUTION

2 (a)

STEPS

1. Work on each equation so that you have the x and y terms on the left-hand side and the number on the other side.
2. Eliminate either the x's or the y's.
3. Solve for the remaining letter.
4. Substitute into either of the original equations to get the other letter.

$3(x-4) = 4y \Rightarrow 3x - 12 = 4y \Rightarrow 3x - 4y = 12$

1. Eliminate the y's by multiplying Eqn. **(A)** by -2.

$$5x - 2y = 13 \ldots \textbf{(A)} (\times -2)$$
$$3x - 4y = 12 \ldots \textbf{(B)}$$

2. Add the two equations together.

$$-10x + 4y = -26$$
$$3x - 4y = 12$$
$$-7x = -14$$

3. Solve the resulting equation for x.

$$-7x = -14 \Rightarrow x = 2$$

4. Substitute the value of x into Eqn. **(B)**

$$3(2) - 4y = 12$$
$$\Rightarrow 6 - 4y = 12$$
$$\Rightarrow -4y = 6 \Rightarrow y = -\tfrac{3}{2}$$

ANSWER: $x = 2,\ y = -\frac{3}{2}$

2 (b)

$$\frac{a-b+1}{a+b+1}=\frac{(\frac{1}{8})-(2)+1}{(\frac{1}{8})+(2)+1}=\frac{\frac{1}{8}-\frac{16}{8}+\frac{8}{8}}{\frac{1}{8}+\frac{16}{8}+\frac{8}{8}}=\frac{-\frac{7}{8}}{\frac{25}{8}}=-\frac{7}{8}\times\frac{8}{25}=-\frac{7}{25}$$

2 (c) (i)

$$\sqrt{125}=125^{\frac{1}{2}}=(5^3)^{\frac{1}{2}}=5^{\frac{3}{2}}$$

2 (c) (ii)

1. $\dfrac{5^{2x+1}}{\sqrt{5}}=\left(\dfrac{1}{\sqrt{125}}\right)^3$

$$\Rightarrow \frac{5^{2x+1}}{5^{\frac{1}{2}}}=\left(\frac{1}{5^{\frac{3}{2}}}\right)^3$$

$$\Rightarrow 5^{2x+\frac{1}{2}}=(5^{-\frac{3}{2}})^3$$

$$\Rightarrow 5^{2x+\frac{1}{2}}=5^{-\frac{9}{2}}$$

2. $\Rightarrow 2x+\frac{1}{2}=-\frac{9}{2}$

3. $\Rightarrow 2x=-\frac{9}{2}-\frac{1}{2}$

$$\Rightarrow 2x=-\frac{10}{2}=-5$$

$$\Rightarrow x=-\frac{5}{2}$$

POWER RULES

1. $a^m\times a^n=a^{m+n}$	**Ex.** $x^3\times x^2=x^5$
2. $\dfrac{a^m}{a^n}=a^{m-n}$	**Ex.** $\dfrac{x^5}{x^3}=x^2$
3. $a^0=1$	**Ex.** $5^0=1$
4. $a^{-n}=\dfrac{1}{a^n}$	**Ex.** $x^{-3}=\dfrac{1}{x^3}$
5. $(a^m)^n=a^{mn}$	**Ex.** $(x^3)^2=x^6$
6. $\sqrt{a}=a^{\frac{1}{2}}$	**Ex.** $\sqrt{9}=9^{\frac{1}{2}}=3$

STEPS

1. Tidy up both sides using the Power Rules until you have the same base and nothing else on both sides.
2. Put the powers equal to one another.
3. Solve for the variable (usually x.)

3 (a) Express p in terms of q and t when

$$q+\frac{p}{5t}=3,\ t\neq 0.$$

(b) (i) If $(x-2)$ is a factor of $3x^3+x^2+kx+6$, find the value of k.

(ii) Write down an equation which has three roots of value -3, 1 and 5.

(c) (i) Write $\dfrac{1}{x+1}+\dfrac{2}{x-3}$ as a single fraction where $x\neq -1$ and $x\neq 3$.

(ii) Hence, or otherwise, find, correct to one place of decimals, the two solutions of

$$\frac{1}{x+1}+\frac{2}{x-3}=1,\ x\neq -1,\ x\neq 3.$$

SOLUTION

3 (a)

$q+\dfrac{p}{5t}=3$ [Multiply each term by $5t$.]

$$\Rightarrow 5tq+p=15t$$

$$\Rightarrow p=15t-5tq$$

3 (b) (i)

The factor theorem states that:

If $(x-k)$ is a factor of $f(x)$ then k is a root of $f(x) = 0$, i.e. $f(k) = 0$ and vice versa.

If $(x-2)$ is a factor of $f(x)=3x^3+x^2+kx+6 \Rightarrow f(2)=0.$

$\therefore f(2)=3(2)^3+(2)^2+k(2)+6=0$

$\Rightarrow 24+4+2k+6=0 \Rightarrow 2k+34=0$

$\Rightarrow 2k=-34 \Rightarrow k=-17$

3 (b) (ii)

-3 is a root $\Rightarrow (x+3)$ is a factor.

1 is a root $\Rightarrow (x-1)$ is a factor.

5 is a root $\Rightarrow (x-5)$ is a factor.

Cubic equation: $\Rightarrow (x+3)(x-1)(x-5)=0$

$\Rightarrow (x+3)(x^2-6x+5)=0$

$\Rightarrow x^3-6x^2+5x+3x^2-18x+15=0$

$\Rightarrow x^3-3x^2-13x+15=0$

3 (c) (i)

$\frac{1}{x+1}+\frac{2}{x-3}$ [Get the common denominator which is $(x+1)(x-3)$.]

$=\frac{1(x-3)+2(x+1)}{(x+1)(x-3)}$ [Multiply out the top and tidy up.]

$=\frac{x-3+2x+2}{(x+1)(x-3)}=\frac{3x-1}{(x+1)(x-3)}$

3 (c) (ii)

$\frac{1}{x+1}+\frac{2}{x-3}=1 \Rightarrow \frac{3x-1}{(x+1)(x-3)}=1$ [Multiply both sides by $(x+1)(x-3)$.]

$\Rightarrow 3x-1=1(x+1)(x-3)$ [Multiply out the brackets.]

$\Rightarrow 3x-1=x^2-2x-3$ [Bring all terms to one side.]

$\Rightarrow 0=x^2-5x-2$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ 2

$a=1$
$b=-5$
$c=-2$

$\therefore x=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(-2)}}{2(1)}=\frac{5\pm\sqrt{25+8}}{2}$

$=\frac{5\pm\sqrt{33}}{2}$

$\therefore x=-0.4,\ 5.4$ [Using calculator.]