ALGEBRA (Q 2 & 3, PAPER 1)

1997

2 (a) Solve for x

$$3(2x-1)=4x.$$

(b) Find the solution set E of $9-2x\geq 7,\ x\in \mathbf{N}$.

Find the solution set H of $\frac{1}{4}x-\frac{1}{3}\leq\frac{5}{12},\ x\in\mathbf{N}$.

Write down the elements of $H\backslash E$.

(c) Simplify

$$\left(\sqrt{x}+\frac{3}{\sqrt{x}}\right)\left(\sqrt{x}-\frac{3}{\sqrt{x}}\right) \text{ where } x>0.$$

Hence solve for x

$$\left(\sqrt{x}+\frac{3}{\sqrt{x}}\right)\left(\sqrt{x}-\frac{3}{\sqrt{x}}\right)=8 \text{ where } x>0.$$

SOLUTION

2 (a)

$3(2x-1)=4x$ [Multiply out the brackets.]

$\Rightarrow 6x-3=4x$ [Bring the x terms to the left and the numbers to the right.]

$\Rightarrow 6x-4x=3$ [Add like terms.]

$\Rightarrow 2x=3$ [Divide both sides by 2.]

$\Rightarrow x=\frac{3}{2}$

2 (b)

$9-2x\geq 7\Rightarrow -2x\geq 7-9$

$\Rightarrow -2x\geq -2$

$\Rightarrow x\leq 1$

$\therefore E=\{0, 1\}$

N: Set of natural numbers. These are whole positive numbers.
N = {0, 1, 2, 3,...}

$H\backslash E$: H less E (The elements in H that are not in E.)

$\frac{1}{4}x-\frac{1}{3}\leq\frac{5}{12}$

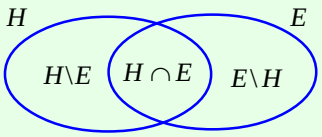
$\Rightarrow 3x-4\leq 12$

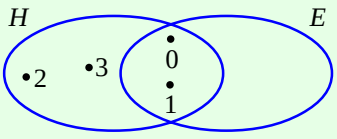
$\Rightarrow 3x\leq 12+4$

$\Rightarrow 3x\leq 16$

$\Rightarrow x\leq\frac{16}{3}$

$\therefore H=\{0, 1, 2, 3\}$





$\therefore H\backslash E=\{2, 3\}$

2 (c)

$\left(\sqrt{x}+\frac{3}{\sqrt{x}}\right)\left(\sqrt{x}-\frac{3}{\sqrt{x}}\right)$ [Multiply each term in the first bracket by each term in the second bracket.]

$=\sqrt{x}\sqrt{x}-\sqrt{x}\frac{3}{\sqrt{x}}-\frac{3}{\sqrt{x}}\sqrt{x}-\frac{3}{\sqrt{x}}\times\frac{3}{\sqrt{x}}$

$(\sqrt{x})^2=\sqrt{x}\times\sqrt{x}=x$

$=x-\frac{9}{x}$

$\left(\sqrt{x}+\frac{3}{\sqrt{x}}\right)\left(\sqrt{x}-\frac{3}{\sqrt{x}}\right)=8\Rightarrow x-\frac{9}{x}=8$ [Multiply each term by x.]

$\Rightarrow x^2-9=8x$

$\Rightarrow x^2-8x-9=0$ [Factorise the quadratic.]

$\Rightarrow (x-9)(x+1)=0$

$\therefore x=-1,\ 9$

3 (a) Express p in terms of q and t when

$$2p-q=3(p-t).$$

(b) Solve the equation

$$2x^3+3x^2-5x-6=0.$$

(c) Let $f(x)=(2+x)(3-x),\ x\in\mathbf{R}$.
Write down the solutions (roots) of $f(x)=0$.
Let $g(x)=3x-k$.
The equation $f(x)+g(x)=0$ has equal roots. Find the value of k.

SOLUTION

3 (a)

$2p-q=3(p-t)$ [Multiply out the bracket.]

$\Rightarrow 2p-q=3p-3t$ [Bring the p terms to one side and the other terms to the other side.]

$\Rightarrow 3t-q=3p-2p$ [Add the p terms.]

$\Rightarrow 3t-q=p$

3 (b)

Solving cubic equations:

STEPS

1. Guess at a root (unless a root is given) by substituting in numbers $0,\ 1,\ -1,\ 2,\ -2,\ldots$ until you get zero.
2. Using the factor theorem, form a factor from the root.
3. Divide the cubic by the factor to get the quadratic.
4. Solve the quadratic by factorising or using formula **2**.
5. Write down the three roots.

1. $f(1) = 2(1)^3 + 3(1)^2 - 5(1) - 6 = 2 + 3 - 5 - 6 = -6 \neq 0$

$f(-1) = 2(-1)^3 + 3(-1)^2 - 5(-1) - 6 = -2 + 3 + 5 - 6 = 0$

2. $\therefore (x+1)$ is a factor.

3. Divide this factor into the cubic as shown on the right.

$\therefore 2x^3 + 3x^2 - 5x - 6 = (x+1)(2x^2 + x - 6) = 0$

$$
\begin{array}{r|l}
 & 2x^2 + x - 6 \\
\hline
x+1 & 2x^3 + 3x^2 - 5x - 6 \\
 & \mp 2x^3 \mp 2x^2 \\
\hline
 & x^2 - 5x - 6 \\
 & \mp x^2 \mp x \\
\hline
 & -6x - 6 \\
 & \pm 6x \pm 6 \\
\hline
 & 0
\end{array}
$$

4. Factorise the quadratic.

$2x^2 + x - 6 = (2x - 3)(x + 2)$

5. $\therefore 2x^3 + 3x^2 - 5x - 6 = (x+1)(2x-3)(x+2) = 0$

Set each factor equal to zero and solve for x.

$\therefore x = -2, -1, \frac{3}{2}$

3 (c)

$f(x) = (2+x)(3-x)$.

$f(x) = 0 \Rightarrow (2+x)(3-x) = 0$ [Set each factor equal to zero and solve for x.]

$\therefore x = -2, 3$

$g(x) = 3x - k$.

$f(x) + g(x) = 0 \Rightarrow (2+x)(3-x) + 3x - k = 0$

$\Rightarrow 6 + x - x^2 + 3x - k = 0$

$\Rightarrow -x^2 + 4x + (6-k) = 0$

$\Rightarrow x^2 - 4x - (6-k) = 0$

EQUAL ROOTS

The quadratic equation $ax^2 + bx + c = 0$ has equal roots if $b^2 = 4ac$.

$b^2 = 4ac \Rightarrow (-4)^2 = 4(1)(k-6)$

$\Rightarrow 16 = 4k - 24$

$\Rightarrow 40 = 4k \Rightarrow k = 10$

$a = 1$

$b = -4$

$c = k - 6$