ALGEBRA (Q 2 & 3, PAPER 1)

1996

2 (a) Solve

$$2x - y = 7$$
$$x + 2y = 6.$$

(b) Write as a power of 2

(i) 16

(ii) $\sqrt{8}$.

Solve for x the equation

$$2^{2x-1} = \left(\frac{16}{\sqrt{8}}\right)^3.$$

(c) Solve

$$\frac{x-1}{x} - \frac{3x}{x-1} = 2, \ x \neq 0 \ \text{ and } \ x \neq 1.$$

SOLUTION

2 (a)

STEPS

1. Work on each equation so that you have the x and y terms on the left-hand side and the number on the other side.
2. Eliminate either the x's or the y's.
3. Solve for the remaining letter.
4. Substitute into either of the original equations to get the other letter.

1. Eliminate the y's by multiplying Eqn. **(A)** by 2.

$$2x - y = 7 \text{...}(\mathbf{A})(\times 2)$$
$$x + 2y = 6 \text{...}(\mathbf{B})$$

2. Add the two equations together.

$$4x - 2y = 14$$
$$x + 2y = 6$$
$$5x \quad = 20$$

3. Solve the resulting equation for x.

$$5x = 20 \Rightarrow x = 4$$

4. Substitute the value of x into Eqn. **(B)**

$$(4) + 2y = 6$$
$$\Rightarrow 2y = 2$$
$$\Rightarrow y = 1$$

ANSWER: $x = 4$, $y = 1$

2 (b) (i)

$16 = 2^4$

2 (b) (ii)

$\sqrt{8} = 8^{\frac{1}{2}} = (2^3)^{\frac{1}{2}} = 2^{\frac{3}{2}}$

1. $2^{2x-1} = \left(\dfrac{16}{\sqrt{8}}\right)^3 \Rightarrow 2^{2x-1} = \left(\dfrac{2^4}{2^{\frac{3}{2}}}\right)^3$

$\Rightarrow 2^{2x-1} = (2^{\frac{5}{2}})^3$

$\Rightarrow 2^{2x-1} = 2^{\frac{15}{2}}$

2. $\Rightarrow 2x - 1 = \frac{15}{2}$

3. $\Rightarrow 2x = \frac{15}{2} + 1 \Rightarrow 2x = \frac{17}{2}$

$\Rightarrow x = \frac{17}{4}$

2 (c)

$\dfrac{x-1}{x} - \dfrac{3x}{x-1} = 2$ [Multiply each term by $x(x-1)$.]

$\dfrac{(x-1)x(x-1)}{x} - \dfrac{3x(x)(x-1)}{(x-1)} = 2x(x-1)$ [Cancel brackets that are the same.]

$\Rightarrow (x-1)(x-1) - 3x^2 = 2x^2 - 2x$ [Multiply out the brackets.]

$\Rightarrow x^2 - 2x + 1 - 3x^2 = 2x^2 - 2x$ [Bring all terms to the left hand side.]

$\Rightarrow -4x^2 + 1 = 0$ [Multiply across by -1.]

$\Rightarrow 4x^2 - 1 = 0$ [Factorise the quadratic.] $a^2 - b^2 = (a+b)(a-b)$ 1

$\Rightarrow (2x+1)(2x-1) = 0$

$\therefore x = -\frac{1}{2}, \frac{1}{2}$

3 (a) Express q in terms of p and t when

$2(p - 3q) = t.$

(b) Find the roots of the equation

$2x^3 - 5x^2 + x + 2 = 0.$

(c) Let $f(x) = (1-x)(2+x),\ x \in \mathbf{R}$.

Write down the solutions of $f(x) = 0$.

Find the range of values of x for which $f(x) > 0$.

Let $g(x) = f(x) - f(x+1)$.

Express $g(x)$ in the form $ax + b,\ a, b \in \mathbf{R}$.

Find the solution set of $g(x) < 0$.

SOLUTION

3 (a)

$2(p-3q)=t$ [Multiply out the bracket.]

$\Rightarrow 2p-6q=t$ [Isolate the q term.]

$\Rightarrow 2p-t=6q$ [Divide both sides by 6.]

$$\Rightarrow \frac{2p-t}{6}=q$$

3 (b)

STEPS
1. Guess at a root (unless a root is given) by substituting in numbers $0, 1, -1, 2, -2, \ldots$ until you get zero.
2. Using the factor theorem, form a factor from the root.
3. Divide the cubic by the factor to get the quadratic.
4. Solve the quadratic by factorising or using formula **2**.
5. Write down the three roots.

1. $f(x)=2x^3-5x^2+x+2 \Rightarrow f(1)=2(1)^3-5(1)^2+(1)+2$

$\Rightarrow f(1)=2-5+1+2=0$

2. $\therefore (x-1)$ is a factor.

3. Divide this factor into the cubic as shown to the right.

$\therefore 2x^3-5x^2+x+2=(x-1)(2x^2-3x-2)=0$

4. Factorise the quadratic.

$2x^2-3x-2=(2x+1)(x-2)$

5. $\therefore 2x^3-5x^2+x+2=(x-1)(2x+1)(x-2)=0$

Set each factor equal to zero and solve for x.

$\therefore x=-\frac{1}{2}, 1, 2$

$$
\begin{array}{r|l}
 & 2x^2-3x-2 \\
\hline
x-1 & 2x^3-5x^2+x+2 \\
 & \mp 2x^3 \pm 2x^2 \\
\hline
 & -3x^2+x+2 \\
 & \mp 3x^2 \mp 3x \\
\hline
 & -2x+2 \\
 & \pm 2x \mp 2 \\
\hline
 & 0
\end{array}
$$

3 (c)

$f(x)=(1-x)(2+x)$

$f(x)=0 \Rightarrow (1-x)(2+x)=0$ [Set each factor equal to zero and solve for x.]

$\therefore x=-2, 1$

To solve quadratic inequalities, you need to sketch the graph of the quadratic function.

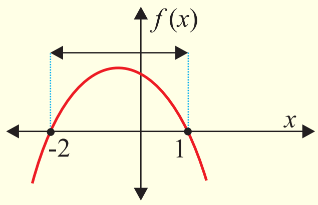
STEPS
1. Find the roots of the quadratic equation: $ax^2+bx+c=0$. These are the places where the curve crosses the x-axis.
2. Sketch the graph. It is either $\cup$ shaped or $\cap$ shaped.
3. Use the graph to solve the inequality.
   $y=f(x)>0$ is above the x-axis.
   $y=f(x)<0$ is below the x-axis.

$f(x)>0 \Rightarrow (1-x)(2+x)>0$ [Multiply out the brackets.]

$\Rightarrow 2+x-x^2>0$

The part of the graph above the *x*-axis satisfies the inequality.

$\therefore -2 < x < 1$



$g(x) = f(x) - f(x+1)$

$= 2 - x - x^2 - [2 - (x+1) - (x+1)^2]$

$= 2 - x - x^2 - [2 - (x+1) - (x^2 + 2x + 1)]$

$= 2 - x - x^2 - [2 - x - 1 - x^2 - 2x - 1]$

$= 2 - x - x^2 - [-3x - x^2]$

$= 2 - x - x^2 + 3x + x^2$

$= 2x + 2$

$g(x) < 0 \Rightarrow 2x + 2 < 0$

$\Rightarrow x + 1 < 0$

$\Rightarrow x < -1$