QUICK OVERVIEW OF ARITHMETIC

PROFIT AND LOSS

Profit = Selling Price − Cost Price
Loss = Cost Price − Selling Price

$$\% \text{ Profit} = \frac{\text{Profit}}{\text{Cost Price}} \times 100; \quad \% \text{ Loss} = \frac{\text{Loss}}{\text{Cost Price}} \times 100 \quad \ldots\ldots\ (1)$$

PERCENTAGE ERROR

$$\% \text{ Error} = \frac{\text{Absolute Error}}{\text{True Value}} \times 100\% \quad \ldots\ldots\ (2)$$

INTEREST

If the sum of money P is invested for n years at the rate per annum of R% which remains unchanged for each year then the amount at the end of n years is:

$$A = P\left(1 + \frac{R}{100}\right)^n \quad \ldots\ldots\ (3)$$

INCOME TAX

Net Tax = Gross Tax − Tax Credits
Take home pay = Gross Income − Net Tax

Higher Rate 41%
Standard Rate 20%

← Standard Rate Cut-Off point

SPEED

$$v = \frac{s}{t} \quad \ldots\ldots\ (4)$$

QUICK OVERVIEW OF ALGEBRA

DIFFERENCE OF 2 SQUARES

$$a^2 - b^2 = (a+b)(a-b) \quad \ldots\ldots\ (1)$$

FORMULA FOR SOLVING QUADRATIC EQUATIONS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \ldots\ldots\ (2)$$

EQUAL ROOTS

The quadratic equation $ax^2 + bx + c = 0$ has equal roots if $b^2 = 4ac$.

FACTOR THEOREM

If $(x-k)$ is a factor of $f(x)$ then k is a root of $f(x) = 0$, i.e. $f(k) = 0$ and vice versa.

POWER RULES

1. $a^m \times a^n = a^{m+n}$	**Ex.** $x^3 \times x^2 = x^5$
2. $\dfrac{a^m}{a^n} = a^{m-n}$	**Ex.** $\dfrac{x^5}{x^3} = x^2$
3. $a^0 = 1$	**Ex.** $5^0 = 1$
4. $a^{-n} = \dfrac{1}{a^n}$	**Ex.** $x^{-3} = \dfrac{1}{x^3}$
5. $(a^m)^n = a^{mn}$	**Ex.** $(x^3)^2 = x^6$
6. $\sqrt{a} = a^{\frac{1}{2}}$	**Ex.** $\sqrt{9} = 9^{\frac{1}{2}} = 3$

QUICK OVERVIEW OF COMPLEX NUMBERS

Powers of i

$i = \sqrt{-1} = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^{power} = i^{\text{remainder when } power \text{ is divided by 4}}$

When you see a power of i, divide the power by 4 and take the remainder. Now use the table on the left to write your answer.

CONJUGATE

$$z = a + bi \Rightarrow \overline{z} = a - bi \quad \ldots\ldots\ (1)$$

If you multiply $a + ib$ by its conjugate $a - ib$ you get $a^2 + b^2$.

MODULUS

$$z = a + bi \Rightarrow |z| = \sqrt{a^2 + b^2} \quad \ldots\ldots\ (2)$$

EQUATIONS

For all equations you can equate (set equal) the real parts and the imaginary parts.

QUADRATIC EQUATIONS

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \ldots\ldots\ (3)$$

If $a + bi$ is a root of a quadratic equation with all real coefficients, then its conjugate, $a - bi$, is also a root.

QUICK OVERVIEW OF SEQUENCES & SERIES

$S_n - S_{n-1} = T_n$ .... **1** Ex. $S_{10} - S_9 = T_{10}$

General term of an arithmetic sequence: $T_n = a + (n-1)d$ **2**

Ex. The fifty-sixth term of an arithmetic sequence: $T_{56} = a + 55d$

Summing formula of an arithmetic sequence: $S_n = \frac{n}{2}[2a + (n-1)d]$ **3**

TEST FOR AN ARITHMETIC SEQUENCE

Any term – Previous term = $T_n - T_{n-1} = \text{Constant } (d)$

General term of a geometric sequence: $T_n = ar^{n-1}$ **4**

Ex. The tenth term of an geometric sequence: $T_{10} = ar^9$

Summing formula of a geometric sequence: $S_n = \dfrac{a(1-r^n)}{(1-r)}$ **5**

TEST FOR A GEOMETRIC SEQUENCE

Any term ÷ Previous term $= \dfrac{T_n}{T_{n-1}} = \text{Constant } (r)$

QUICK OVERVIEW OF DIFFERENTIATION & FUNCTIONS

BASIC RULE OF DIFFERENTIATION: $y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$ **1**

THE PRODUCT RULE: If $y = u \times v$ then: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ **2**

THE QUOTIENT RULE: If $y = \frac{u}{v}$ then: $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ **3**

THE CHAIN RULE: $y = (u)^n \Rightarrow \frac{dy}{dx} = n(u)^{n-1} \times \frac{du}{dx}$ ... **1**

EQUATION OF THE LINE (TANGENTS): $y - y_1 = m(x - x_1)$ **4**

INCREASING AND DECREASING CURVES

A curve is increasing if its slope is positive, i.e. $\frac{dy}{dx} > 0$.

A curve is decreasing if its slope is negative, i.e. $\frac{dy}{dx} < 0$.

.... **5**

TURNING POINTS

Turning Point $\Rightarrow \frac{dy}{dx} = 0$ **6**

To find the turning points set $\frac{dy}{dx} = 0$ and solve for x.

LOCAL MAXIMUM AND MINIMUM

Local Maximum: $\left(\frac{d^2y}{dx^2}\right)_{TP} < 0$

Local Minimum: $\left(\frac{d^2y}{dx^2}\right)_{TP} > 0$

.... **7**

RATE OF CHANGE FORMULAE

$v = \frac{ds}{dt}$ **8**

$a = \frac{dv}{dt}$ **9**

QUICK OVERVIEW OF AREAS & VOLUMES

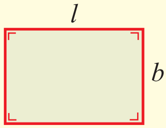
Many of the formulae for this chapter appear on pages 6 & 7 of the Tables and are reproduced at the end of this book.

AREAS AND PERIMETERS OF REGULAR SHAPES

RECTANGLE

$A = l \times b$

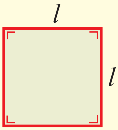
$P = 2l + 2b = 2(l + b)$ **1**



SQUARE

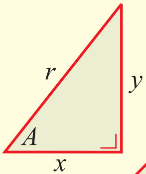
$A = l \times l = l^2$

$P = 4l$ **2**

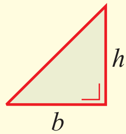


RIGHT-ANGLED TRIANGLES

$x^2 + y^2 = r^2$ **3**

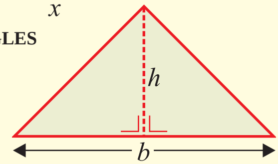


$A = \frac{1}{2}bh$ **4**

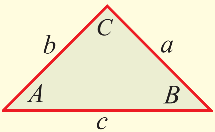


NON RIGHT-ANGLED TRIANGLES

$A = \frac{1}{2}bh$ **4**

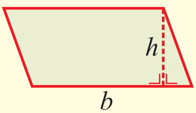


$A = \frac{1}{2}ab\sin C$ **5**



PARALLELOGRAM

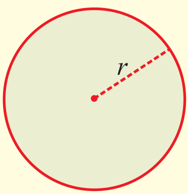
$A = b \times h$ **6**



CIRCLE

$L = 2\pi r$ **7**

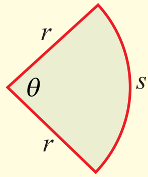
$A = \pi r^2$ **8**



SECTOR

$$s = 2\pi r \times \frac{\theta}{360^\circ}$$ 9

$$A = \pi r^2 \times \frac{\theta}{360^\circ}$$ 10



AREA OF IRREGULAR SHAPES (SIMPSON'S RULE)

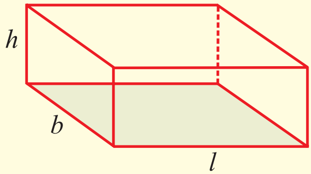
$$A \approx \tfrac{h}{3}\,[(\text{First} + \text{Last}) + 4(\text{Evens}) + 2(\text{Remaining Odds})]$$ 11

VOLUMES AND SURFACE AREAS OF REGULAR SHAPES

RECTANGULAR SOLID

$V = l \times b \times h$

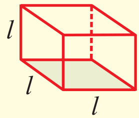
Surface Area $A = 2(lb + bh + lh)$ 12



CUBE

$V = l^3$

Surface Area $A = 6l^2$ 13

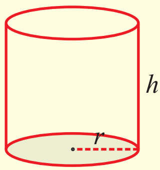


CYLINDER

$V = \pi r^2 h$

Curved SA: $A = 2\pi rh$

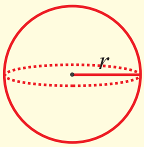
Total SA: $A = 2\pi rh + 2\pi r^2$ 14



SPHERE

$V = \tfrac{4}{3}\pi r^3$

Curved SA: $A = 4\pi r^2$ 15

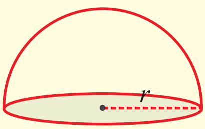


HEMISPHERE

$V = \tfrac{2}{3}\pi r^3$

Curved SA: $A = 2\pi r^2$

Total SA: $A = 3\pi r^2$ 16

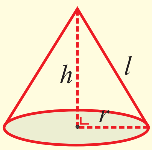


CONE

$V = \tfrac{1}{3}\pi r^2 h$

Curved SA: $A = \pi rl$

Total SA: $A = \pi rl + \pi r^2$ 17



QUICK OVERVIEW OF THE LINE

DISTANCE OF A LINE SEGMENT

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{.......} \quad (1)$$

MIDPOINT OF A LINE SEGMENT

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \quad \text{.......} \quad (2)$$

SLOPE OF A LINE

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{.......} \quad (3)$$

Parallel lines have the same slope.

$$K \parallel L \Rightarrow m_1 = m_2$$

Two lines are perpendicular if the product of their slopes is -1.

$$K \perp L \Rightarrow m_1 \times m_2 = -1$$

EQUATION OF A LINE

$$y - y_1 = m(x - x_1) \quad \text{.......} \quad (4)$$

READING A SLOPE, m, FROM THE EQUATION OF A LINE, $ax + by + c = 0$:

$$m = -\left(\frac{a}{b} \right) \quad \text{.......} \quad (5)$$

IS A POINT ON A LINE?

To show a point is on a line, put the point into the equation.

INTERCEPTS OF THE AXES

To find the x-intercept: Put $y = 0$.
To find the y-intercept: Put $x = 0$.

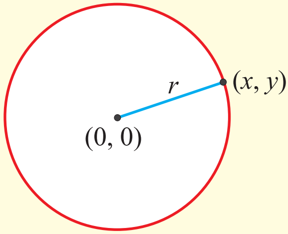
AREA OF A TRIANGLE

$$A = \tfrac{1}{2}\left| x_1 y_2 - x_2 y_1 \right| \quad \text{.......} \quad (6)$$

QUICK OVERVIEW OF THE CIRCLE

Circle C with centre (0, 0), radius r.

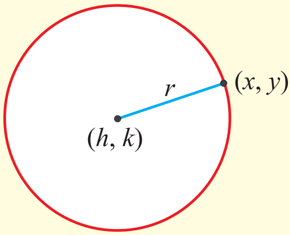
$x^2 + y^2 = r^2$ **1**



IS A POINT ON A CIRCLE, INSIDE A CIRCLE OR OUTSIDE A CIRCLE?
Substitute the point into the circle.
On the circle: Both sides are equal.
Inside the circle: The left hand side is less than the right hand side.
Outside the circle: The left hand side is greater than the right hand side.

Circle C with centre (h, k), radius r.

$(x-h)^2 + (y-k)^2 = r^2$ **2**



TO SHOW A LINE SEGMENT IS A DIAMETER OF A CIRCLE:
The midpoint of a diameter is the centre of a circle.

TANGENTS

You need to use the equation of a line: $(y - y_1) = m(x - x_1)$

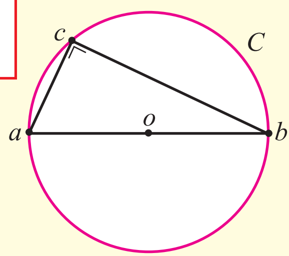
PROOF THAT A LINE IS A TANGENT TO A CIRCLE: When you solve the quadratic only one answer is obtained, i.e. one point of contact.

INTERCEPTS WITH THE AXES

TO FIND OUT WHERE THE CIRCLE, C, CROSSES THE x-AXIS:
Set $y = 0$ in the circle equation.
TO FIND OUT WHERE THE CIRCLE, C, CROSSES THE y-AXIS:
Set $x = 0$ in the circle equation.

RIGHT-ANGLES TRIANGLES INSIDE CIRCLES

Slope of ac × Slope of $bc = -1 \Rightarrow ac \perp bc$



Quick Overview of Geometry

*There are ten theorems you need to learn. Theorem **1** has two deductions.*

Theorem 1: The sum of the degree measures of the interior angles of a triangle is 180^o.

> **Deduction** 1: The degree measure of the exterior angle of a triangle is equal to the sum of the two remote interior angles.
>
> **Deduction** 2: An exterior angle of a triangle is greater than either of the two remote (opposite) interior angles.

Theorem 2: The opposite sides of a parallelogram have equal lengths.

Theorem 3: If three parallel lines make intercepts of equal length on a transversal, then they will make intercepts of equal lengths on any other transversal.

Theorem 4: A line which is parallel to one side of a triangle, and cuts a second side, will cut the third side in the same proportion as the second.

Theorem 5: If the three angles of one triangle have a degree measure equal respectively to the degree measure of the angles of a second triangle then the lengths of the corresponding sides of the two triangles are proportional.

Theorem 6: In a right-angled triangle the square of the length of the side opposite to the right-angle is equal to the sum of the squares of the lengths of the other two sides.

THEOREM 7: (Converse of Pythagoras) If the square of the length of one side of a triangle is equal to the sum of the squares of the lengths of the other two sides then the triangle has a right angle and this is opposite the longest side.

THEOREM 8: The products of the lengths of the sides of a triangle by the corresponding altitudes are equal.

THEOREM 9: If the lengths of a triangle are unequal, then the degree measures of the angles opposite to them are unequal, with the greater angle opposite to the longer side.

THEOREM 10: The sum of the lengths of any two sides of a triangle is greater than that of the third side.

ENLARGEMENTS

$$\text{Scale factor } k = \frac{|\text{Image length}|}{|\text{Object length}|} \quad \ldots\ldots\ (1)$$

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} \quad \ldots\ldots\ (2)$$

QUICK OVERVIEW OF TRIGONOMETRY

RIGHT-ANGLED TRIANGLES

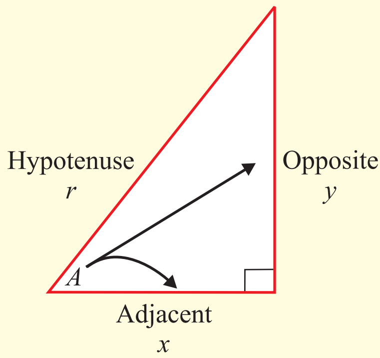
Pythagoras: $x^2 + y^2 = r^2$ 1

Area: $A = \frac{1}{2}bh$ 2

$\cos A = \frac{x}{r} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ 3

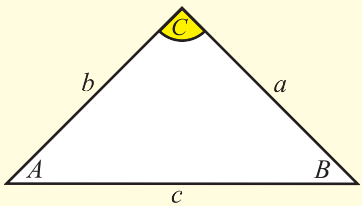
$\sin A = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}}$ 4

$\tan A = \frac{y}{x} = \frac{\text{Opposite}}{\text{Adjacent}}$ 5



NON RIGHT-ANGLED TRIANGLES

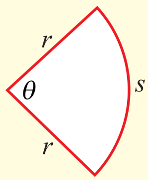
Area: $A = \frac{1}{2}ab\sin C$ 6



SECTOR FORMULAE

Length of arc: $s = 2\pi r \times \frac{\theta}{360^\circ}$ 7

Area of sector: $A = \pi r^2 \times \frac{\theta}{360^\circ}$ 8



SINE RULE

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ 9

THE COSINE RULE

$a^2 = b^2 + c^2 - 2bc\cos A$ 10

QUICK OVERVIEW OF COUNTING & PROBABILITY

COMBINATIONS

The number of selections of n different objects taking r at a time $= {}^nC_r = \binom{n}{r}$ 1

PERMUTATIONS

The number of arrangements of n different objects taking r at a time with no repeats $= {}^nP_r$ 2

The number of arrangements of n different objects all taken, no repeats = $n!$ 3

PROBABILITY

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}}$$ 4

RULE 1

$p(A \text{ and then } B) = p(A) \times p(B)$ 5

RULE 2

$p(A \text{ or } B) = p(A) + p(B) - p(A \text{ and } B)$ 6

RULE 3

$p(A$ happens at least once$) = 1 - p(A$ does not happen at all$)$ 7

QUICK OVERVIEW OF STATISTICS

MEAN OF A LIST OF NUMBERS

$$\bar{x} = \frac{x_1 + x_2 + \dots\dots\dots\dots + x_N}{N} = \frac{\text{Sum of the Numbers}}{\text{Number of Numbers}} = \frac{\sum x}{N} \quad \dots\dots\dots \quad (1)$$

MEAN OF A FREQUENCY DISTRIBUTION

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + \dots\dots + f_N x_N}{f_1 + f_2 + \dots\dots\dots + f_N} = \frac{\sum fx}{\sum f} \quad \dots\dots\dots \quad (2)$$

WEIGHTED MEAN

$$\bar{w} = \frac{\sum wx}{\sum w} \quad \dots\dots\dots \quad (3)$$

STANDARD DEVIATION OF A LIST OF NUMBERS

$$\sigma = \sqrt{\frac{\text{Sum of (Deviations)}^2}{\text{Number of numbers}}} = \sqrt{\frac{\sum d^2}{N}} \quad \dots\dots\dots \quad (4)$$

The deviation, d, is given by the formula: $d = (x - \bar{x}) = (\text{Number} - \text{Mean})$.

STANDARD DEVIATION OF A FREQUENCY DISTRIBUTION

$$\sigma = \sqrt{\frac{\sum fd^2}{\sum f}} \quad \dots\dots\dots \quad (5)$$

HISTOGRAMS

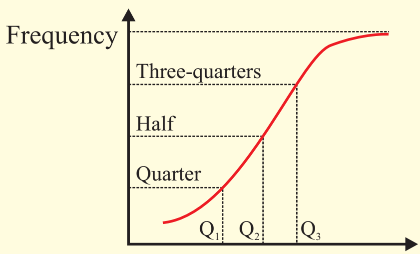
Histograms are ways of showing information in which the area of a box is equal to the frequency.

$$\text{Area (Frequency)} = \text{Base} \times \text{Height}$$

CUMULATIVE FREQUENCY (OGIVE)

Q_1: Lower Quartile
Q_2: Median
Q_3: Upper Quartile

Interquartile range $= Q_3 - Q_1$

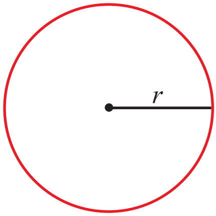


THE TABLES

You are allowed to use the official Department of Education table book in the exam hall. There is a lot of information in this book that you do not need. The information on the following pages has been extracted from the official table book and is exactly what you need for the Leaving Cert. Ordinary Level Maths papers.

PAGE 6 & 7 OF THE TABLES

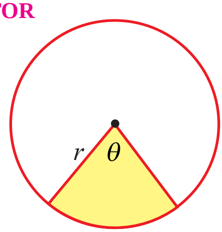
CIRCLE



Length $= 2\pi r$

Area $= \pi r^2$

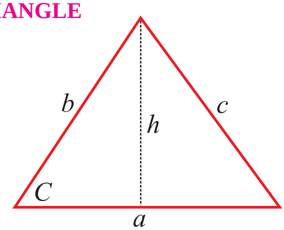
SECTOR



Length $= r\theta$ (θ in radians)

Area $= \frac{1}{2} r^2\theta$ (θ in radians)

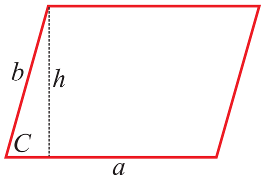
TRIANGLE



Area $= \frac{1}{2} ah$

Area $= \frac{1}{2} ab \sin C$

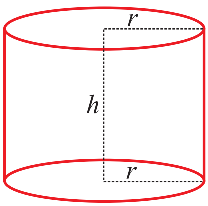
PARALLELOGRAM



Area $= ah$

Area $= ab \sin C$

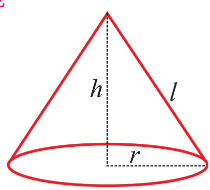
CYLINDER



Area of curved surface $= 2\pi rh$

Volume $= \pi r^2 h$

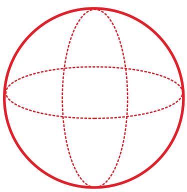
CONE



Curved surface area $= \pi rl$

Volume $= \frac{1}{3}\pi r^2 h$

SPHERE



Area of surface $= 4\pi r^2$

Volume $= \frac{4}{3}\pi r^3$

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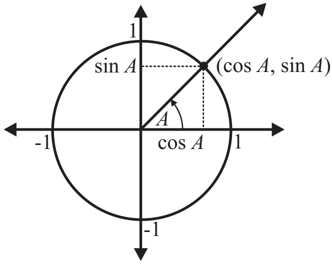
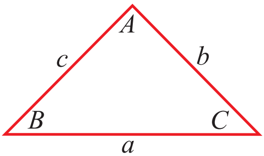
$\cos^2 A + \sin^2 A = 1$

$\tan A = \dfrac{\sin A}{\cos A}$



A	0	π	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$
A	0^o	180^o	90^o	60^o	45^o	30^o
$\cos A$	1	-1	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\sin A$	0	0	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan A$	0	0	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$

Use the Sine and Cosine rules to solve triangles.



Sine formula: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine formula: $a^2 = b^2 + c^2 - 2bc\cos A$

Compound Angle formulae

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

The formulae for $\cos(A-B)$ and $\sin(A-B)$ can be obtained by changing the signs in these formulae.

These formulae are obtained by replacing B by A in the compound angle formulae.

$\cos 2A = \cos^2 A - \sin^2 A$

$\sin 2A = 2\sin A\cos A$

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DIFFERENTIATION

$f(x)$	$f'(x) \equiv \frac{d}{dx}[f(x)]$
x^n	nx^{n-1}

Products and Quotients:

$$y = uv; \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$y = \frac{u}{v}; \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$