QUESTION 8 (75 MARKS)

QUESTION 8 (a)

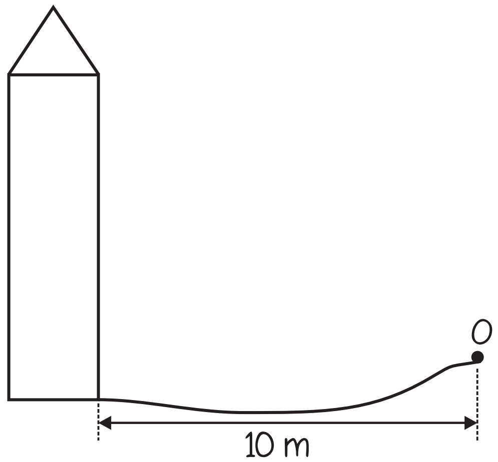
A tower that is part of a hotel has a square base of side 4 metres and a roof in the form of a pyramid. The owners plan to cover the roof with copper. To find the amount of copper needed, they need to know the total area of the roof.

A surveyor stands 10 metres from the tower, measured horizontally, and makes observations of angles of elevation from the point O as follows:

The angle of elevation of the top of the roof is 46°.

The angle of elevation of the closest point at the bottom of the roof is 42°.

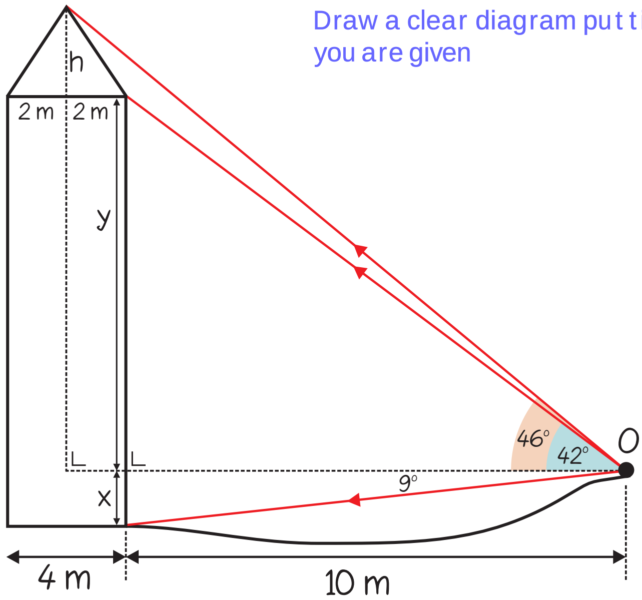
The angle of depression of the closest point at the bottom of the tower is 9°.

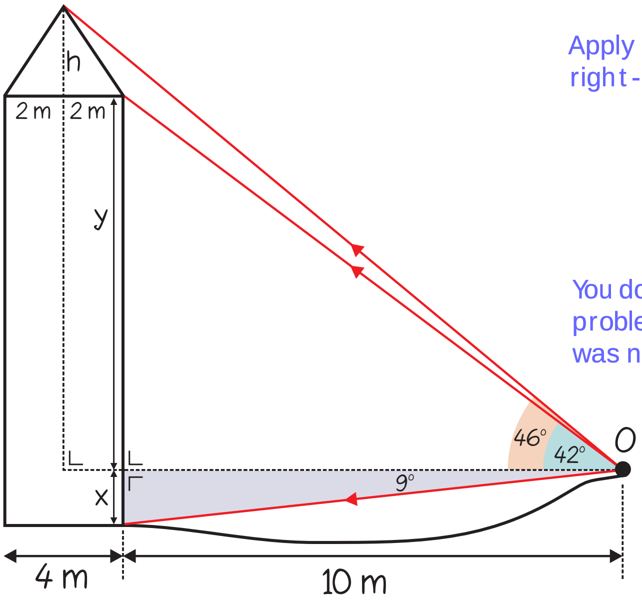


QUESTION 8 (a) (i)

Find the vertical height of the roof.

SOLUTION



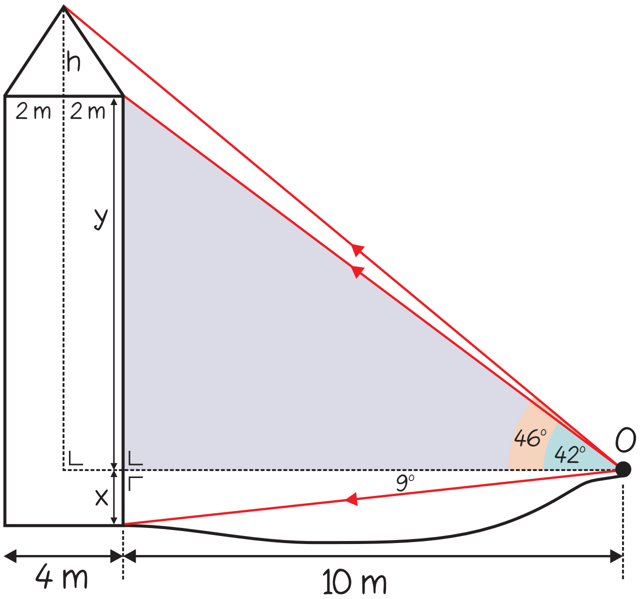


Apply the tan function to each of the purple right-angle triangles.

$$\tan 9^\circ = \frac{x}{10}$$

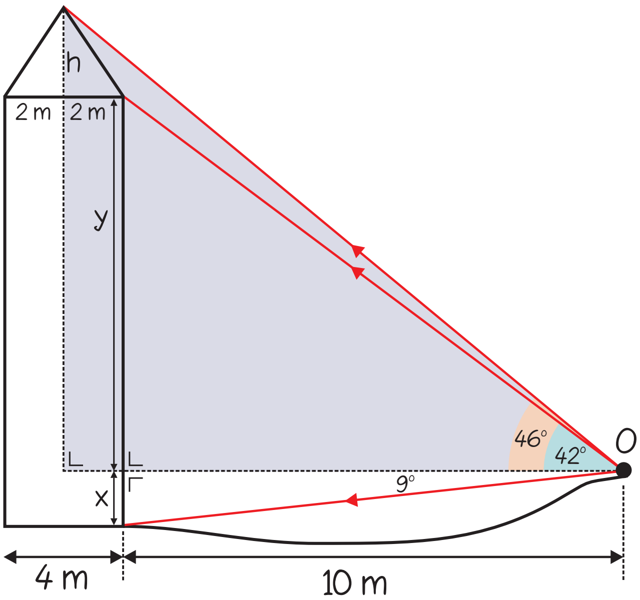
$$\therefore x = 10\tan 9^\circ$$

You do not need to find x for the rest of the problem. Giving you the angle of depression was not necessary to solve this problem.



$$\tan 42^\circ = \frac{y}{10}$$

$$\therefore y = 10\tan 42^\circ$$



$$\tan 46^\circ = \frac{h+y}{12}$$

$$\therefore h + y = 12\tan 46^\circ$$

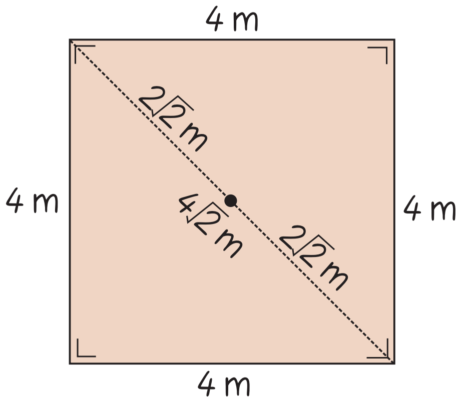
$$h = 12\tan 46^\circ - y$$

$$h = 12\tan 46^\circ - 10\tan 42^\circ = 3.422\,\text{m}$$

QUESTION 8 (a) (ii)

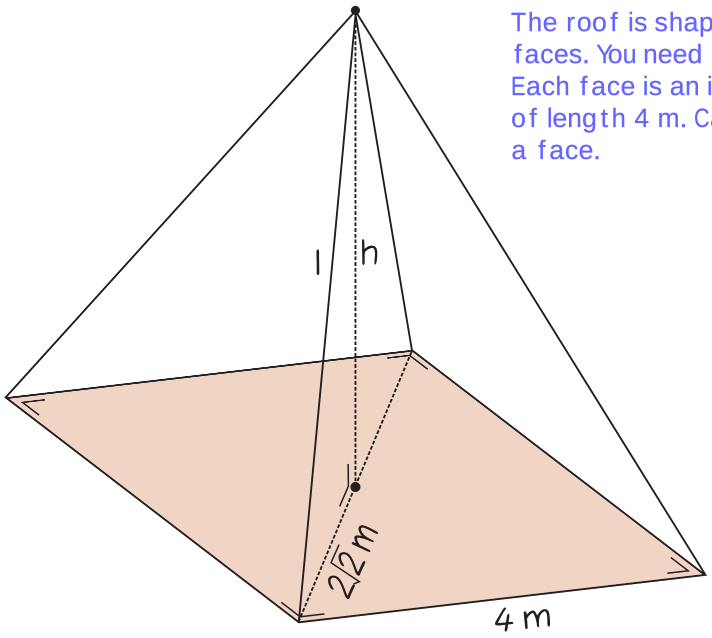
Find the total area of the roof.

SOLUTION

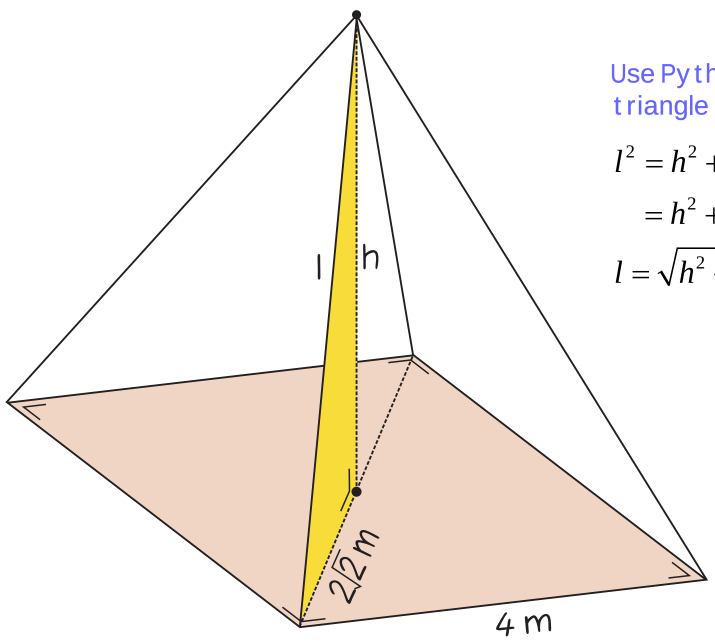


The roof has a square base. Call the length of the diagonal of this base d. Use Pythagoras to find d.

$$d^2 = 4^2 + 4^2$$
$$= 16 + 16$$
$$= 32$$
$$d = \sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2} \text{ m}$$

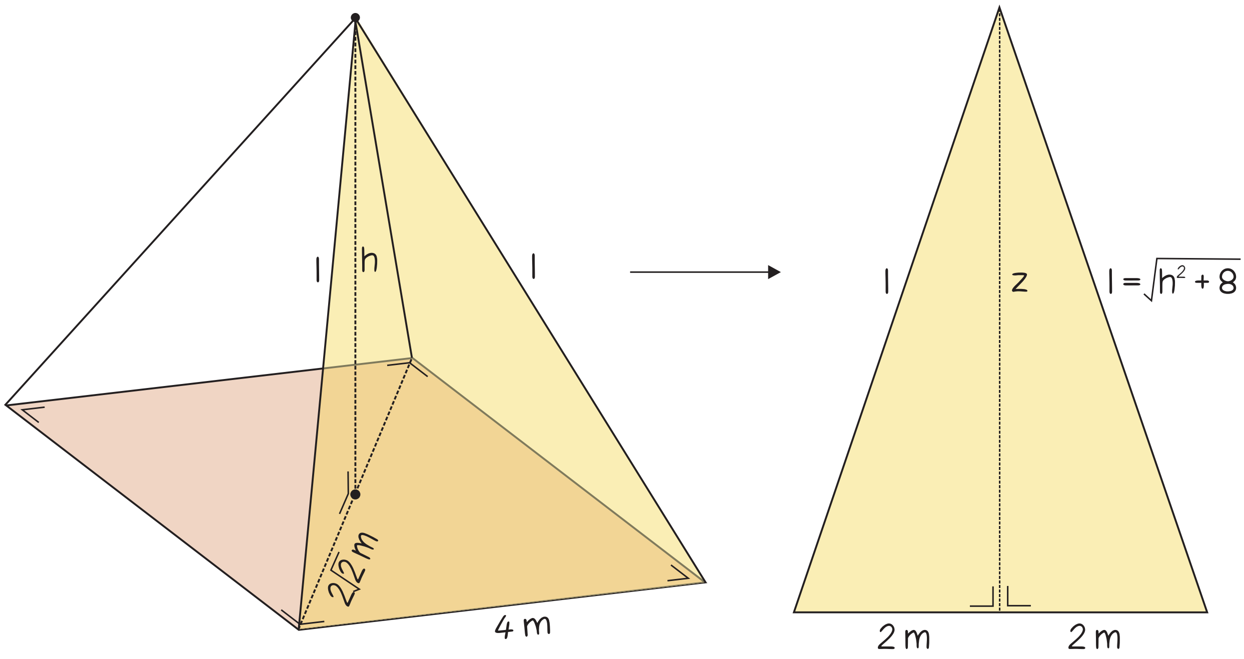


The roof is shaped as a pyramid. It has 4 faces. You need to find the area of the faces. Each face is an isosceles triangle with a base of length 4 m. Call l the length of the side of a face.



Use Pythagoras on the yellow right-angled triangle to find l.

$$l^2 = h^2 + (2\sqrt{2})^2$$
$$= h^2 + 8$$
$$l = \sqrt{h^2 + 8}$$



Lift out one of the faces of the roof. It divides into two right-angled triangles. Call z the height of this triangle. Use Pythagoras to find z.

$$z^2 + 2^2 = (\sqrt{h^2 + 8})^2$$
$$z^2 + 4 = h^2 + 8$$
$$z^2 = h^2 + 4$$
$$z = \sqrt{h^2 + 4}$$

Area $A = \frac{1}{2}(4)\sqrt{h^2+4} = 2\sqrt{h^2+4}\ \text{m}^2$ The area is half the base by the perpendicular height.

Total Roof area = $8\sqrt{h^2+4}\ \text{m}^2 = 8\sqrt{3.422^2+4}\ \text{m}^2 = 31.71\ \text{m}^2$ Multiply the area by 4 to get the total area of the roof.

QUESTION 8 (a) (iii)

If all of the angles observed are subject to a possible error of ±1°, find the range of possible areas for the roof.

SOLUTION

A possible error of ±1° means the 42° angle of elevation can be anywhere between 41° and 43° and the 46° angle of elevation can be anywhere between 45° and 47°.

Therefore the roof will have a maximum height h_1 if the angles of elevations are 41° and 47° and a minimum hight h_2 if the angles of elevations are 43° and 45°.

Maximum height $h_1 = 12\tan 47^\circ - 10\tan 41^\circ = 4.176\ \text{m}$

Minimum height $h_2 = 12\tan 45^\circ - 10\tan 43^\circ = 2.675\ \text{m}$

Maximum roof area $A_1 = 8\sqrt{h_1^2+4} = 8\sqrt{4.176^2+4} = 37.04\ \text{m}^2$

Minimum roof area $A_2 = 8\sqrt{h_2^2+4} = 8\sqrt{2.675^2+4} = 26.72\ \text{m}^2$

QUESTION 8 (b)

Twenty five students each measure and record a particular angle of elevation, in degrees, each using his or her own home-made clinometer. The results are as follows:

24	20	22	15	70
15	16	15	16	15
18	16	21	21	73
16	20	12	18	20
18	18	14	22	18

QUESTION 8 (b) (i)

Find what you consider to be the best estimate of the true value of the angle, explaining your reasoning.

SOLUTION

There are a number of possibilities for finding the best estimate:

POSSIBILITY NUMBER 1:

24	20	22	15	70
15	16	15	16	15
18	16	21	21	73
16	20	12	18	20
18	18	14	22	18

The two results, 70 and 73, are outliers. They are obviously wrong results. We can decide to ignore them by deciding that the students taking these readings got it completely wrong. We can then take the mean of the other results and use this as our best estimate.

$$\bar{x} = \frac{410}{23} = 17.83$$

POSSIBILITY NUMBER 2:

24	20	22	15	20
15	16	15	16	15
18	16	21	21	17
16	20	12	18	20
18	18	14	22	18

The two results, 70 and 73, are outliers. We may decide that the students taking these readings carried out the correct procedure but read the complementary angle, $(90^\circ - \theta)$, on their clinometers. We can correct these results and then take the mean of all the results and use this as our best estimate.

$$\bar{x} = \frac{447}{25} = 17.88$$

POSSIBILITY NUMBER 3:

12	14	15	15	15
15	16	16	16	16
18	18	18	18	18
20	20	20	21	21
22	22	24	70	73

Use the median as the best estimate by arranging the results in order. The median is the thirteenth result irrespective of whether you correct the outliers, leave them there or ignore them. Medians are less influenced by outliers.

Median = 18

QUESTION 8 (b) (ii)

Based on previous experience, a teacher has claimed that, in these circumstances, half of all students will measure the angle correctly to within two degrees. Taking these students to be a simple random sample, and assuming the true value of the angle is the one you calculated in part **(i)**, is there sufficient evidence to reject the teacher's claim at the 5% level of significance?

SOLUTION

Hypothesis H: Half of all students will measure the angle correctly to within 2^o.

95% margin of error for sample of size 25: $\frac{1}{\sqrt{25}}=\frac{1}{5}=0.2$

Reject the hypothesis H if sample proportion lies outside 0.5 ± 0.2, i.e. between 0.3 and 0.7. Let's say possibility Number 2 was the one you chose.

POSSIBILITY NUMBER 2:

24	20	22	15	20
15	16	15	16	15
18	16	21	21	17
16	20	12	18	20
18	18	14	22	18

Mean $\bar{x}=17.88$ is the best estimate. The number of students within 2^o of the best estimate, i.e. between 15.88 and 19.88, is 10. They are highlighted above.

Sample Proportion: $\frac{10}{25}=0.4$

The sample proportion, 0.4, lies between 0.3 and 0.7. Therefore, do not reject the hypothesis H. Therefore, there is not sufficient evidence to reject the teacher's claim at the 5% level of significance.