

MATHS PROBLEM NO. 2

An aeroplane engine fails with probability x . A three-engined plane needs 2 or 3 engines for a successful flight.

Find

- (i) the probability of a successful flight on a one-engined plane in terms of x ,
- (ii) the probability of a successful flight in a three-engined plane,
- (iii) the range of values of x for which a one-engined plane is safer than a three-engined plane.



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SOLUTION

- (i) One engined plane:

$$p(\text{Failed flight}) = x$$

$$p(\text{Successful flight}) = (1 - x)$$

- (ii) Three-engined flight:

$$p(\text{Successful flight}) = p(3 \text{ engines working}) \text{ or } p(2 \text{ engines working and one failing})$$

$$= (1 - x) \times (1 - x) \times (1 - x) + (1 - x) \times (1 - x) \times x \times \frac{3!}{2!}$$

$$= (1 - x)(1 - x)(1 - x) + 3x(1 - x)(1 - x)$$

$$= (1 - x)^2[(1 - x) + 3x]$$

$$= (1 - x)^2[1 - x + 3x]$$

$$= (1 - x)^2[2x + 1]$$

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SOLUTION

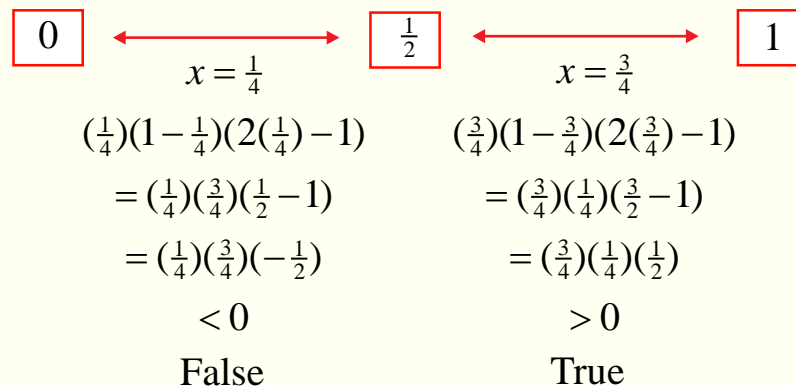
(iii) $p(\text{Successful flight in a one-engined plane}) > p(\text{Successful flight in a three-engine flight})$

$$\begin{aligned}
 (1-x) &> (1-x)^2(2x+1) \\
 (1-x) - (1-x)^2(2x+1) &> 0 \\
 (1-x)[1 - (1-x)(2x+1)] &> 0 \\
 (1-x)[1 - (2x+1-2x^2-x)] &> 0 \\
 (1-x)[1 - (x+1-2x^2)] &> 0 \\
 (1-x)[1-x-1+2x^2] &> 0 \\
 (1-x)[2x^2-x] &> 0 \\
 (1-x)[x(2x-1)] &> 0 \\
 x(1-x)(2x-1) &> 0
 \end{aligned}$$

Solve the equality:

$$\begin{aligned}
 x(1-x)(2x-1) &= 0 \\
 \therefore x &= 0, 1, \frac{1}{2}
 \end{aligned}$$

NOTE: There are no solutions less than 0 or greater than 1 as the probability of an event must be between 0 and 1.



ANSWER: $\frac{1}{2} < x < 1$