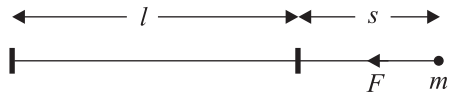
11. Simple Harmonic Motion

Simple Harmonic Motion (SHM) is an example of oscillating or periodic motion.

Hooke's Law: For an elastic string or spring the restoring force is directly proportional to the displacement (extension/compression).



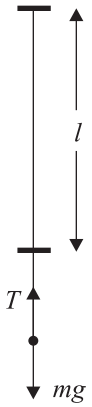
l: Unstretched length (natural length)
s: Displacement (Extension/compression)

$$F = -ks \Rightarrow ma = -ks$$

$$a = -\tfrac{k}{m}s \Rightarrow a = -\omega^2 s \text{ where } \omega = \sqrt{\tfrac{k}{m}}.$$

The negative sign indicates that the restoring force is always in the opposite direction to the displacement.
k: Elastic constant. It's unit is N m^{-1}. It's value depends on the material and the geometry of the spring.

For a vertical spring/string: $T = mg = kd \Rightarrow k = \frac{mg}{d}$



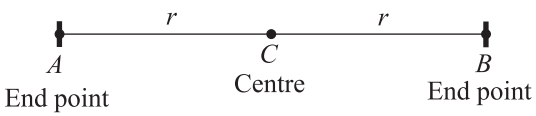
Definition: **Hooke's Law** states that for an elastic string or spring the restoring force is directly proportional to the displacement.

Motion under the influence of the type of force described by Hooke's law is called SHM and its equation is:

$$a = -\omega^2 s$$

a: Acceleration (m s^{-2})
s: Displacement (m)
ω: Constant

Definitions



SHM: Motion in a path in which the acceleration is proportional to the displacement of the mass from the equilibrium position and is in the opposite direction.

Displacement, s: The directed distance of an object from the equilibrium position [centre, C]

Amplitude, r: The maximum distance from the centre. [$|AC| = |CB| = r$]

Period, P: The time for a complete cycle. [Time to go from A to C to B and back to a again]

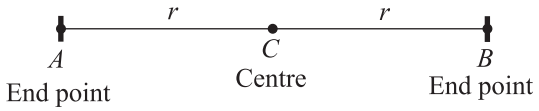
Frequency, f: The number of cycles per second.

The periodic time, P, is the time for the motion to undergo a complete cycle.

$$P = \frac{1}{f} = \frac{2\pi}{\omega}$$

P: Periodic Time (s)
f: Frequency (Hz)
ω: Angular velocity (rad s^{-1})

The maximum displacement is called the amplitude, r. Consider an object oscillating from A to B about its equilibrium position C. Its maximum velocity is at C and its velocity is zero at A and B. However, its acceleration in a maximum at A and B because the maximum force is acting on the object. Its acceleration is zero at C because the force acting on the object is zero.



s:	$-r$	0	r
v:	0	$v_{\text{Max.}} = r\omega$	0
a:	$a_{\text{Max.}} = \omega^2 r$	0	$a_{\text{Max.}} = \omega^2 r$
F:	$F_{\text{Max.}} = m\omega^2 r$	0	$F_{\text{Max.}} = m\omega^2 r$

Example: A particle is undergoing SHM. If its acceleration is 8 cm s^{-1} at 2 cm find (a) ω, (b) P, (c) maximum acceleration, (d) maximum speed if its amplitude is 4 cm.

SOLUTION

$a = 8\,\text{cm}\,\text{s}^{-1} = 0{\cdot}08\,\text{m}\,\text{s}^{-1}$
$s = 2\,\text{cm} = 0{\cdot}02\,\text{m}$
r = 4 cm = 0·04 m

(a) $a = \omega^2 s \Rightarrow 0{\cdot}08 = \omega^2 \times 0{\cdot}02$
$\therefore \omega = 2$

(b) $P = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi\,\text{s}$

(c) $a_{\text{Max.}} = \omega^2 r = 4 \times 0{\cdot}04 = 0{\cdot}16\,\text{m}\,\text{s}^{-2}$

(d) $v_{\text{Max.}} = r\omega = 0{\cdot}04 \times 2 = 0{\cdot}08\,\text{m}\,\text{s}^{-1}$

Example: A particle of mass 300 g executes SHM on the end of an elastic string with a period of 20 s. Find the elastic constant.

SOLUTION

m = 300 g = 0·3 kg
P = 20 s

$$P = 20\,\text{s} = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{\pi}{10} = \sqrt{\frac{k}{m}}$$

$$\Rightarrow \frac{\pi^2}{100} = \frac{k}{0{\cdot}3}$$

$$\therefore k = \frac{0{\cdot}3\pi^2}{100} = 2{\cdot}95 \times 10^{-2}\,\text{N}\,\text{m}^{-1}$$

EVERYDAY EXAMPLES OF SHM

A **pendulum** consists of a point mass (the bob) suspended by a light string. It oscillates in a vertical plane under the force of gravity.
An oscillating **spring**.
The prongs of a vibrating **tuning fork**.

Numerical Problems

1. Acceleration of a particle under SHM is 2 m s^{-2} when its displacement is 10 cm. Find its period.

2. The acceleration of a particle under SHM is 2·5 m s^{-2} when its displacement is 15 cm. What is its period?

3. A particle executes SHM of period 2 s. What is its acceleration when 20 cm from the equilibrium position?

4. A body executes SHM with an amplitude of 200 mm. The maximum force is 0·064 N. Calculate the maximum speed and the period if its mass is 200 g.

5. A particle moves with SHM of amplitude 0·05 m and a period of 12 s. Find
(a) the maximum speed,
(b) the maximum acceleration.

6. A particle undergoing SHM of amplitude 0·04 m and period 10 s. Find
(a) the maximum speed,
(b) the maximum acceleration.

7. A bob of a simple pendulum undergoes SHM with amplitude 16 cm and period 4 s. The mass is 0·4 kg. Find
(a) maximum speed,
(b) maximum kinetic energy.

8. A simple pendulum moves with SHM of amplitude 8 cm and period 2 s. Its mass is 0·5 kg. Find
(a) the maximum speed,
(b) the maximum kinetic energy.

9. A particle executes SHM of period 3 s. What is its acceleration when 20 cm from its equilibrium position?

10. A body of mass 400 g undergoes SHM with amplitude 40 mm. The maximum force is 0·08 N. Find
(a) maximum speed,
(b) period.

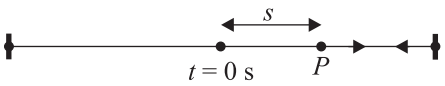
11. A spring is extended 4 cm when a 4 kg mass is hung from it. Find the elastic constant if g = 9·8 m s^{-2}.

12. A particle undergoing SHM of amplitude has a frequency of 2 Hz. Find
(a) period,
(b) acceleration when 3 cm from centre.

13. A spring oscillates with amplitude 3·2 cm. If the elastic constant is 240 N m^{-1} and mass is 0·4 kg find
(a) ω,
(b) P,
(c) acceleration at 2 cm from the centre.

14. A spring of elastic constant 10 N m^{-1} has a 500 g load hanging from it. Find the periodic time. Find the extension of the spring. If it is pulled down 15 cm below this position and released from rest, how long does it take to reach the equilibrium position again?
(g = 9·8 m s^{-2})

15. A particle executes SHM and passes a particular point with acceleration 4 m s^{-2} at intervals of 2 s and 6 s alternately. Calculate the displacement of this point from the equilibrium rest position.



16. A particle executes SHM with a frequency of 40 Hz. What is its period?

17. A spring has an elastic constant of 12 N m^{-1}. What mass would be suspended to give a period of π s.

18. For a string/spring show that $P^2 = \dfrac{4\pi^2 m}{k}$.

Answers

1. 1·4 s

2. 1·54 s

3. 0·246 m s^{-2}

4. 0·25 m s^{-1}, 5 s

5. 0·026 m s^{-1}, 0·0137 m s^{-2}

6. (a) 0·025 m s^{-1}, (b) 0·0158 m s^{-2}

7. (a) 0·25 m s^{-1}, (b) 0·0125 J

8. (a) 0·25 m s^{-1}, (b) 0·0156 J

9. 0·876 m s^{-2}

10. (a) 0·09 m s^{-1}, (b) 2·8 s

11. 980 N m^{-1}

12. (a) 0·5 s, 4·73 m s^{-2}

13. (a) 18·7 s^{-1}, (b) 0·336 s, (c) 7 m s^{-2}

14. 1·4 s, 0·49 m, 0·35 s

15. 6·5 m

16. 0·025 s

17. 3 kg