1 • Horizontal Linear Motion

In mechanics there are three fundamental quantities: Length (distance), mass and time. The units in which these quantities are measured are called fundamental units. In the SI system they are:
Length: metres (m)
Mass: kilograms (kg)
Time: seconds (s)
All other quantities in mechanics are derived from some combination of these quantities.

Definitions
Displacement (s): The distance a body is in a certain direction from a fixed point. **Unit**: m
Velocity (v): The rate of change of displacement. **Unit**: m s^{-1}
Acceleration (a): The rate of change of velocity. **Unit**: m s^{-2}

Equations of uniformly accelerated motion (UAM)

$$a = \frac{\text{Change in velocity}}{\text{Time}} = \frac{v-u}{t} \Rightarrow v = u + at$$

$\boxed{v = u + at}$ **Eqn**. 1

$$\text{Average velocity} = \frac{s}{t} = \frac{(u+v)}{2} \Rightarrow s = \tfrac{1}{2}(u+v)t$$

$\boxed{s = \tfrac{1}{2}(u+v)t}$ **Eqn**. 2

Substituting 1 into 2 gives:

$$s = \frac{(u+u+at)t}{2} = \frac{2ut + at^2}{2} = ut + \tfrac{1}{2}at^2$$

$\boxed{s = ut + \tfrac{1}{2}at^2}$ **Eqn**. 3

Squaring 1 gives:

$$v^2 = u^2 + 2uat + a^2t^2 \Rightarrow v^2 = u^2 + 2a(ut + \tfrac{1}{2}at^2) = u^2 + 2as$$

$\boxed{v^2 = u^2 + 2as}$ **Eqn**. 4

Rewrite Equation 1 ($u = v - at$) and substitute into 2 gives:

$$s = \frac{(v - at + v)t}{2} = \frac{2vt - at^2}{2} = vt - \tfrac{1}{2}at^2$$

$\boxed{s = vt - \tfrac{1}{2}at^2}$ **Eqn**. 5

Symbols and units	**Signs**
u = initial velocity (m s^{-1})	s, u, v + right
v = final velocity (m s^{-1})	s, u, v – left
a = acceleration/deceleration (m s^{-2})	a + acceleration
t = time (s)	a – deceleration
s = displacement (m)	

All displacements must be measured from where the body is at $t = 0$.
These equations only hold for uniformly accelerated motion (a constant) in a straight line.

Remember for any motion:

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

PROBLEM SOLVING TECHNIQUES

1. Make a list of the numbers given and assign each the correct symbol (u, v, a, s, t).
2. Make sure the units are correct.
3. Choose the correct formula (1 of 5):

$$v = u + at \quad (\text{No } s)$$

$$s = \tfrac{1}{2}(u + v)t \quad (\text{No } a)$$

$$s = ut + \tfrac{1}{2}at^2 \quad (\text{No } v)$$

$$v^2 = u^2 + 2as \quad (\text{No } t)$$

$$s = vt - \tfrac{1}{2}at^2 \quad (\text{No } u)$$

4. Solve the equations.

Example: A body starts with an initial velocity of 4 ms^{-1}. It achieves a velocity of 10 m s^{-1} after travelling 3 m. What is its acceleration?

SOLUTION

$u = +4$ ms^{-1}
$v = +10$ ms^{-1}
$s = +3$ m
$a = ?$

No t. Use $v^2 = u^2 + 2as \Rightarrow 100 = 16 + 2a(3)$

$\Rightarrow 84 = 6a$

$\therefore a = 14 \text{ m s}^{-2}$

Example: A body achieves a velocity of 16 ms^{-1} in 4 seconds with an acceleration of 2 ms^{-2}. How far has it travelled? What is its initial velocity?

SOLUTION

$v = +16$ ms^{-1}
$t = 4$ s
$a = 2$ m s^{-2}
$s = ?$

No u. Use $s = vt - \tfrac{1}{2}at^2 \Rightarrow s = (16)(4) - \tfrac{1}{2}(2)(16) = 64 - 16$

$\therefore s = 48 \text{ m}$

$v = u + at \Rightarrow 16 = u + (2)(4)$

$\therefore u = 8 \text{ m s}^{-1}$

Example: A body starts with an initial velocity of 8 m s^{-1} and a deceleration of 2 m s^{-2}. At what times is its displacement 15 m?

SOLUTION

u = 8 m s^{-1}

a = –2 m s^{-2}

t = ?

s = 15 m

No v. Use $s = ut + \frac{1}{2}at^2 \Rightarrow 15 = 8t + \frac{1}{2}(-2)t^2$

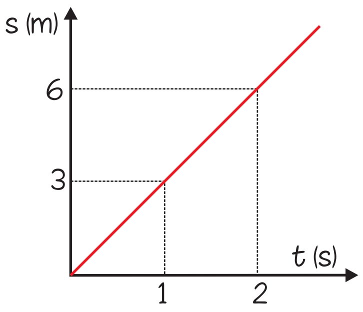
$\Rightarrow t^2 - 8t + 15 = 0 \Rightarrow (t-3)(t-5) = 0$

$\therefore t = 3$ s, 5 s

1.2 GRAPHS

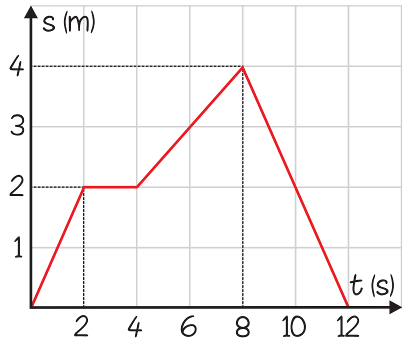
[A] DISPLACEMENT-TIME GRAPHS

This is a graph of displacement s against time t.



The slope = 3 = constant = v,
i.e. the body is moving at a steady velocity of 3 m s^{-1}.

The slope at any point of a displacement-time graph is the velocity.



From 0 – 2 s the velocity is 1 m s^{-1}. From 2 – 4 s the velocity is zero. From 4 – 8 s the velocity is 0.5 m s^{-1}. From 8 – 12 s the velocity is –1 m s^{-1}.

[B] **VELOCITY-TIME GRAPHS**
This is a graph of velocity against time.

From 0 – 4 s the acceleration is the slope of *oa*

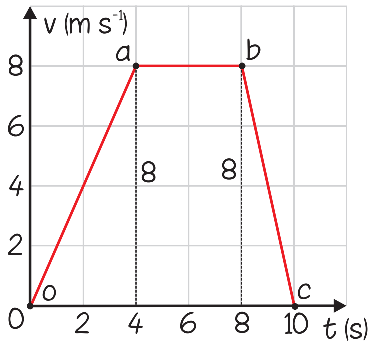
$$=\frac{8\text{ m s}^{-1}}{4\text{ s}}=2\text{ m s}^{-2}.$$

From 4 – 8 s the acceleration is zero, i.e. the body is moving at a steady speed.
From 8 – 10 s the deceleration is the slope of *bc*

$$=\frac{8\text{ m s}^{-1}}{2\text{ s}}=4\text{ m s}^{-2}.$$

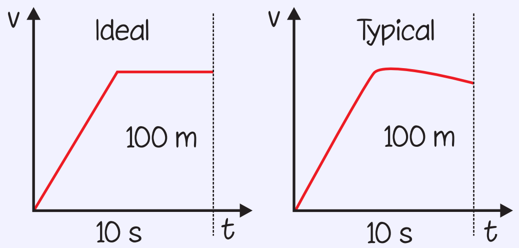
The distance travelled = area of the trapezium *oabc*

$$\tfrac{1}{2}(10+4)8=56\text{ m}.$$



The slope at any point of a velocity-time graph is the acceleration and the area under the curve is the distance travelled.

ATHLETICS: The *v*-*t* profile of a 100 m sprinter is interesting. Ideally she should accelerate out of the blocks at her maximum acceleration up to her maximum speed and then maintain this all the way to the tape.



STOPPING DISTANCE (d):

(i) The stopping distance, d, for a vehicle after the brakes of a car have been applied depends on the initial speed of the car and the deceleration $\left(d=\frac{u^2}{2a}\right)$. It also depends on the condition of the road and of the car.

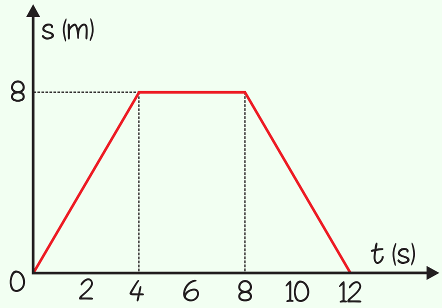
(ii) The actual distance depends on the reaction time of the driver (0.2 s – 0.6 s) and the rate at which the brakes are applied by him.

NOTE:

(i) Area of a triangle = $\frac{1}{2}$ Base Height

(ii) Area of a trapezium = $\frac{1}{2}$ (Sum of parallel sides) Height

Example: Using the displacement-time curve find
(i) the displacement after 4 s, 6 s, 8 s, 12 s,
(ii) velocity during the first 4 s, velocity during the next 4 s and the velocity during the last 4 s,
(iii) the total distance travelled over 12 s,
(iv) average speed.



Solution

(i) After 4 s, $s = 8$ m
After 6 s, $s = 8$ m
After 8 s, $s = 8$ m
After 12 s, $s = 0$ m

(ii) For the first 4 s: $v = \frac{s}{t} = \frac{8-0}{4-0} = 2 \text{ m s}^{-1}$

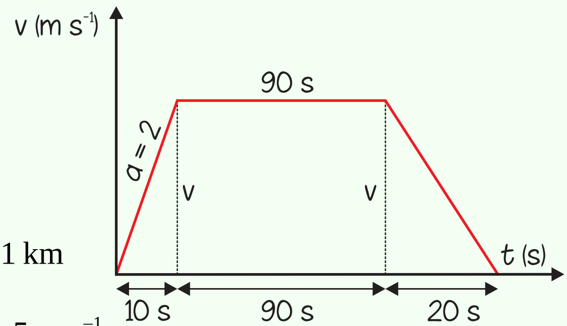
For the next 4 s: $v = \frac{s}{t} = \frac{8-8}{8-4} = 0 \text{ m s}^{-1}$

For the last 4 s: $v = \frac{s}{t} = \frac{0-8}{12-8} = -2 \text{ m s}^{-1}$

(iii) Total distance = 16 m

(iv) Average speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{16}{12} = \frac{4}{3} = 1{\cdot}33 \text{ m s}^{-1}$

Example: A train starts from rest and accelerates at 2 m s^{-2} for 10 s. It maintains the speed so reached for 90 s and then decelerates to rest in 20 s. Draw a velocity-time curve. From it find:
(i) the maximum velocity,
(ii) the total distance travelled,
(iii) the average speed for the journey.



Solution

(i) $a = 2 = \frac{v}{10} \Rightarrow v = 20 \text{ m s}^{-1}$

(ii) $s = \frac{1}{2}(120+90)\times 20 = 10(210) = 2100 \text{ m} = 2{\cdot}1 \text{ km}$

(iii) Average speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{2100}{120} = 17{\cdot}5 \text{ m s}^{-1}$

Numerical Problems

Uniformly Accelerated Motion (UAM)

1. (a) Initially at rest, $a = 6$ m s^{-2}, $t = 4$ s. Find the displacement.

(b) Initial velocity = 7 m s^{-1}, $a = 5$ m s^{-2}, $t = 4$ s. Find the final velocity.

(c) Initial velocity = 2 m s^{-1}, final velocity = 7 m s^{-1}, $t = 10$ s. Find the displacement.

(d) Final velocity = 14 m s^{-1}, $a = 3$ m s^{-2}, $t = 3$ s. Find the initial velocity.

(e) Displacement = 40 m, final velocity = 6 m s^{-1}, $t = 8$ s. Find the initial velocity.

(f) $a = 1$ m s^{-2}, final velocity = 36 km h^{-1}, $t = 2$ s. Find the initial velocity.

(g) Deceleration = 4 m s^{-2}, final velocity = 9 m s^{-1}, $t = 3$s. Find the displacement.

(h) Displacement = 60 m, initial velocity = 4 m s^{-1}, final velocity = 8 m s^{-1}. Find the time.

(i) Initial velocity = 10 m s^{-1}, $a = 2$ m s^{-2}, $s = 24$ m. Find the time.

(j) Displacement = 51 m, initial velocity = 20 m s^{-1}, $a = -2$ m s^{-2}. Find the final velocity.

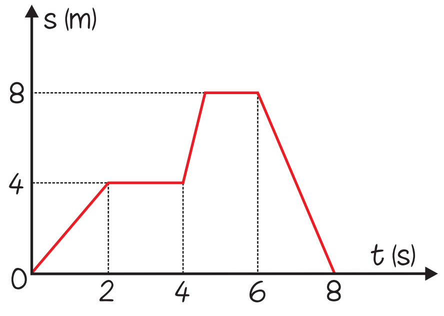
2. A car is travelling at 162 km h^{-1}. The brakes are applied and the car decelerates to 36 km/h in 14 s. What is the deceleration in m s^{-2}.

3. The driver of a car slams on his brakes when he sees a tree blocking the road. The car slows uniformly with a deceleration of 5·3 m s^{-2} for 4 s, making skidmarks 61 m long . What is the speed of the car as it hits the tree?

4. A motorist is travelling at 14 m s^{-1} through Yellowstone National Park. He sees a bear 38 m ahead. If the maximum deceleration of the car is 4·8 m s^{-2} what is the maximum reaction time the motorist can allow himself if he is to avoid hitting the bear?

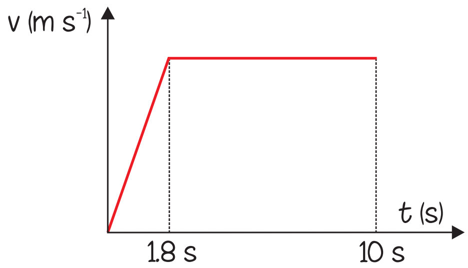
Graphs

5. For the displacement-time curve find
(i) the displacement at times of 2 s, 3 s, 4 s, 5 s, 6 s and 8 s,
(ii) the average velocity from 0 s to 2 s,
(iii) the velocity at times 4 s and 6 s,
(iv) the average speed from times 4 s to 6 s,
(v) the average speed from times 6 s to 8 s,
(vi) the total distance travelled after 8 s,
(vii) the average speed for the whole journey.

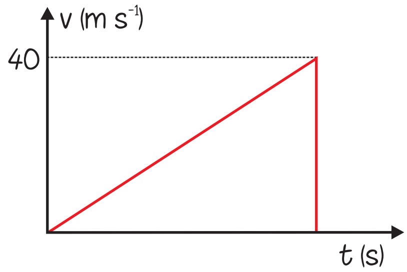


6. A body starts from rest and accelerates to 6 m s^{-1} in 3 seconds. It then travels at this speed for a further 12 s and then decelerates to rest in 2 s. Sketch a *v*-*t* curve and from it find
(i) acceleration,
(ii) deceleration,
(iii) total distance travelled,
(iv) average speed for the journey.

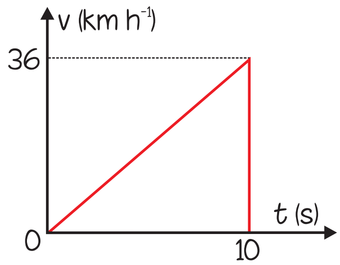
7. A sprinter's velocity-time profile is shown. He reaches his maximum speed in 1·8 s. If the distance covered is 100 m and the total time is 10 s find
(i) the maximum speed,
(ii) the acceleration.



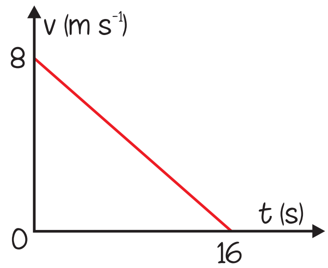
8. Use the following graphs to answer the questions below.
(a) If a = 2 m s^{-2} find
(i) time,
(ii) distance.



(b) Find
(i) the acceleration in m s^{-2},
(ii) distance in m.

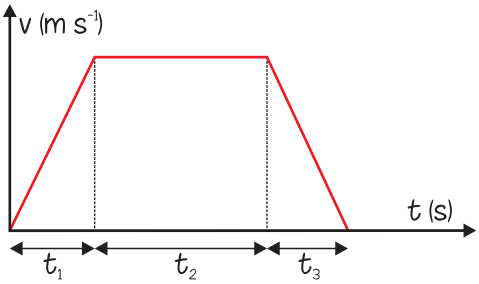


(c) Find
(i) the deceleration,
(ii) the distance.



9. (a) If the acceleration is 6 m s^{-2} and the deceleration is 3 m s^{-2} and the maximum speed is 18 m s^{-1}, find t_1 and t_3. Find t_2 if the total distance is 240 m.

(b) If the acceleration is 2 m s^{-2} and the maximum speed is 10 m s^{-1} and the deceleration is 5 m s^{-2} find t_1 and t_3. If the distance covered is 250 m, find t_2.



ANSWERS

Uniformly accelerated motion (UAM)

1. (a) 48 m
(b) 27 m s^{-1}
(c) 45 m
(d) 5 m s^{-1}
(e) 4 m s^{-1}
(f) 8 m s^{-1}
(g) 45 m
(h) 10 s
(i) 2 s
(j) 14 m s^{-1}

2. 2·5 m s^{-2}

3. 4·65 m s^{-1}

4. 1·26 s

Graphs

5. (i) 4 m, 4 m, 4 m, 8 m, 8 m, 0 m
(ii) 2 m s^{-1}
(iii) 0 m s^{-1}, 0 m s^{-1}
(iv) 2 m s^{-1}
(v) –4 m s^{-1}
(vi) 16 m
(vii) 2 m s^{-1}

6. (i) 2 m s^{-2}
(ii) 3 m s^{-2}
(iii) 87 m
(iv) 5·1 m s^{-1}

7. (i) 11 m s^{-1}
(ii) 6·1 m s^{-2}

8. (a) (i) 20 s, (ii) 400 m
(b) (i) 1 m s^{-2}, (ii) 50 m
(c) (i) –0·5 m s^{-2}, (ii) 64 m

9. (a) 3 s, 6 s, 8·83 s
(b) 5 s, 2 s, 21·5 s