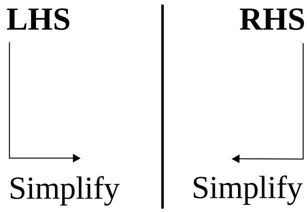
3.3 PROVING IDENTITIES



STEPS

1. Look at the angles. They will be:

Single Angles	Multiple Angles	Compound Angles	Half Angles
SA 1	**MA** 1-2	**CA** 1-4	**HA**

2. Change all trig functions into sin and cos and don't change back.

3. For a half angle (**HA**), $\frac{A}{2}$, call it another letter B immediately and get rid of it as follows: $\frac{A}{2} = B \Rightarrow A = 2B$.

4. Simplify each side using good algebra and **SA** 1, **CA** 1-4 and **MA** 1-2.

5. Keep what God sends you. If you see something on the left that is on the right or vice versa, hold on to it like mad.

6. Never cross over the middle line. Never do the same thing to both sides - these are not equations.

7. Look out for factors (HCF/DOTS/DOTC/SOTC).

NOTES

1. Always change $\tan A$ into $\frac{\sin A}{\cos A}$ and $\cot A$ into $\frac{\cos A}{\sin A}$.

2. Be able to tidy up double decker (DD) fractions by multiplying above and below by the CD of all fractions.

Example: $\dfrac{\frac{\cos A}{\sin A} - \frac{\sin A}{\cos A}}{\frac{1}{\sin A} - \frac{1}{\cos A}}$ is a DD fraction.

Multiply above and below by $\sin A \cos A$.

$$\therefore \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\cos A}}{\frac{1}{\sin A} - \frac{1}{\cos A}} \times \frac{\sin A \cos A}{\sin A \cos A} = \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$

Example 8: Prove $\tan A + \cot A = \dfrac{1}{\sin A \cos A}$.

SOLUTION

Look at the angles. They are all single angles **SA**.

LHS

$$\tan A + \cot A$$
$$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$
$$= \frac{\sin^2 A + \cos^2 A}{\cos A \sin A}$$
$$= \frac{1}{\sin A \cos A}$$

RHS

$$\frac{1}{\sin A \cos A}$$

$\therefore$ LHS = RHS

Example 9: Prove $\dfrac{\sec A - \operatorname{cosec} A}{\tan A - \cot A} = \dfrac{\tan A + \cot A}{\sec A + \operatorname{cosec} A}$.

SOLUTION

The angles are all **SA**.

LHS

$$\frac{\sec A - \operatorname{cosec} A}{\tan A - \cot A}$$
$$= \frac{\frac{1}{\cos A} - \frac{1}{\sin A}}{\frac{\sin A}{\cos A} - \frac{\cos A}{\sin A}} \text{ (DD)}$$
$$= \frac{\sin A - \cos A}{\sin^2 A - \cos^2 A} \text{ — DOTS}$$
$$= \frac{\sin A - \cos A}{(\sin A - \cos A)(\sin A + \cos A)}$$
$$= \frac{1}{\sin A + \cos A}$$

RHS

$$\frac{\tan A + \cot A}{\sec A + \operatorname{cosec} A}$$
$$= \frac{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}{\frac{1}{\cos A} + \frac{1}{\sin A}} \text{ (DD)}$$
$$= \frac{\sin^2 A + \cos^2 A}{\sin A + \cos A}$$
$$= \frac{1}{\sin A + \cos A}$$

$\therefore$ LHS = RHS

Example 10: Prove $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot A + \cot B}$.

SOLUTION

CA on LHS, **SA**'s on RHS

LHS

$\cot(A+B)$

$= \dfrac{\cos(A+B)}{\sin(A+B)}$

$= \dfrac{\cos A\cos B - \sin A \sin B}{\sin A \cos B + \cos A \sin B}$

RHS

$\dfrac{\cot A \cot B - 1}{\cot A + \cot B}$

$= \dfrac{\frac{\cos A}{\sin A}\frac{\cos B}{\sin B} - 1}{\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B}}\left(\dfrac{\sin A \sin B}{\sin A \sin B}\right)$ (DD)

$= \dfrac{\cos A\cos B - \sin A \sin B}{\sin A \cos B + \cos A \sin B}$

$\therefore$ LHS = RHS

Example 11: Prove $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$.

SOLUTION

CA on LHS, **SA**'s on RHS

LHS

$\cos(A+B)\cos(A-B)$

$= (\cos A \cos B - \sin A \sin B)(\cos A \cos B + \sin A \sin B)$

$= \cos^2 A \cos^2 B - \sin^2 A \sin^2 B$

✓ ✗ ✗ ✓

$= \cos^2 A(1 - \sin^2 B) - (1 - \cos^2 A)\sin^2 B$

$= \cos^2 A - \cos^2 A \sin^2 B - \sin^2 B + \cos^2 A \sin^2 B$

$= \cos^2 A - \sin^2 B$ $= RHS$

✓ = Keep it - it's on the RHS

✗ = Get rid of it - it's not on the RHS