6. If $T_n = (5n - 3)2^n$ show that $T_{n+1} - 2T_n = 5.2^{n+1}$.

7. If $S_n = n(n + 1)$ find T_n. Hence, show
$T_{n+3}^2 - T_{n+1}^2 - T_{n+2}^2 + T_n^2 = 16$.

8. If $S_n = \log_{10}(n + 1)$ find T_n and T_{30}.

9. If $T_n = (n - 10)3^n$ show that $T_{n+2} - 6T_{n+1} + 9T_n = 0$.

10. If $T_n = 600(2)^n - 7(5)^n$ show that $T_{n+2} - 7T_{n+1} + 10T_n = 0$.
Find the least value of $n \in N_0$ if $T_n < 0$.

11. If $T_n = (n + 1)(n - 1)!$ show that $nT_n + n! = T_{n+1}$.

12. If $T_n = 5(3)^n + 7(2)^n$ show that $T_{n+2} - 5T_{n+1} + 6T_n = 0$.

1.7 CONVERGENCE/DIVERGENCE OF A SEQUENCE

[A] The Idea: Infinite sequences either approach a definite finite number as you look further and further down the list or they don't, i.e. they approach an infinite value. They are said to be convergent and divergent respectively.

Example: 1.9, 1.99, 1.999, 1.9999, ... gets closer and closer to 2. We say that it is convergent (CGT) to 2.
Example: 1, 3, 5, 7, 9, ... just gets bigger and bigger. We say it is divergent (DGT).

Test for convergence/divergence: Do $\lim_{n \to \infty} T_n$

If $\lim_{n \to \infty} T_n = \text{L}$ (Finite) ...CGT (Convergent)

If $\lim_{n \to \infty} T_n = \infty$ (Infinite) ...DGT (Divergent)

This is called taking an infinity limit.

[B] **Steps to taking an infinity limit:**

STEPS

1. Take out the highest power of the variable from each bracket on the top.
2. Do the same for the bottom.
3. Tidy up the takeouts.
4. Plonk in ∞ for the variable.

SOME IMPORTANT INFINITY LIMITS

1. $\lim_{n\to\infty} \frac{\text{Number}}{n^p} = 0$, for p a whole positive number.

Example: $\lim_{n\to\infty} \frac{8}{(n)^5} = 0$

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2. $\lim_{n\to\infty} (n)^p = \infty$, for p a whole positive number.

Example: $\lim_{n\to\infty} (n)^3 = \infty$

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3. $\lim_{n\to\infty} r^n = 0$ for $-1 < r < 1$

Example: $\lim_{n\to\infty} (\frac{3}{5})^n = 0$

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4. $\lim_{n\to\infty} r^n = \infty$ for $r > 1$, $r < -1$

Example: $\lim_{n\to\infty} (\frac{3}{2})^n = \infty$

NOTES

1. $-1 < r < 1 \Leftrightarrow |r| < 1$
2. $r < -1, r > 1 \Leftrightarrow |r| > 1$

Example 9: Test the sequence $T_n = \dfrac{n^2-4n+7}{(2n+1)(n-3)}$ for convergence.

SOLUTION

$$\lim_{n\to\infty} T_n = \lim_{n\to\infty} \frac{n^2-4n+7}{(2n+1)(n-3)} = \lim_{n\to\infty} \frac{n^2(1-\frac{4}{n}+\frac{7}{n^2})}{n(2+\frac{1}{n})n(1-\frac{3}{n})}$$

$$= \lim_{n\to\infty} \frac{(1-\frac{4}{n}+\frac{7}{n^2})}{(2+\frac{1}{n})(1-\frac{3}{n})} = \frac{(1-0+0)}{(2+0)(1-0)} = \frac{1}{2} \Rightarrow \text{CGT to } \frac{1}{2}$$

Example 10: Test $T_n = \dfrac{(n-2)(5n^2+3n-2)}{(3n-1)(2n+4)}$ for convergence.

SOLUTION

$$\lim_{n\to\infty} T_n = \lim_{n\to\infty} \frac{(n-2)(5n^2+3n-2)}{(3n-1)(2n+4)} = \lim_{n\to\infty} \frac{n(1-\frac{2}{n})n^2(5+\frac{3}{n}-\frac{2}{n^2})}{n(3-\frac{1}{n})n(2+\frac{4}{n})}$$

$$= \lim_{n\to\infty} \frac{n(1-\frac{2}{n})(5+\frac{3}{n}-\frac{2}{n^2})}{(3-\frac{1}{n})(2+\frac{4}{n})} = \frac{\infty(1)(5)}{(3)(2)} = \infty \Rightarrow \text{DGT}$$

Example 11: Test $T_n = \dfrac{4^n}{4^n+5^n}$ for convergence.

SOLUTION

$$T_n = \frac{4^n}{4^n+5^n}$$

$$\lim_{n\to\infty} T_n = \lim_{n\to\infty} \frac{4^n}{5^n((\frac{4}{5})^n+1)} = \lim_{n\to\infty} \frac{(\frac{4}{5})^n}{(\frac{4}{5})^n+1} = \frac{0}{0+1} = 0$$

$\Rightarrow$ CGT to 0.